

Sensing Ionospheric Turbulence Using GNSS

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Motivation

- Turbulence in the troposphere has been studied in the Kolmogorov sense
- $2/3$ $5/3$ power law well verified in time and space by VLBI/GNSS/SAR (other)...
- Turbulence in the ionosphere not so studied (author impression) in Kolmogorov sense
- This study aims to check whether signal coming from GNSS is compatible with Kolmogorov turbulence theory

Kolmogorov Turbulence Theory Prediction

- Kolmogorov turbulence theory predict refractive index that varies with a power law of 2/3

$$\chi(d) = C * d^{2/3}$$

- A radio wave passing through a turbulent medium should have a phase structure function (variogram) that varies with a power law of 5/3

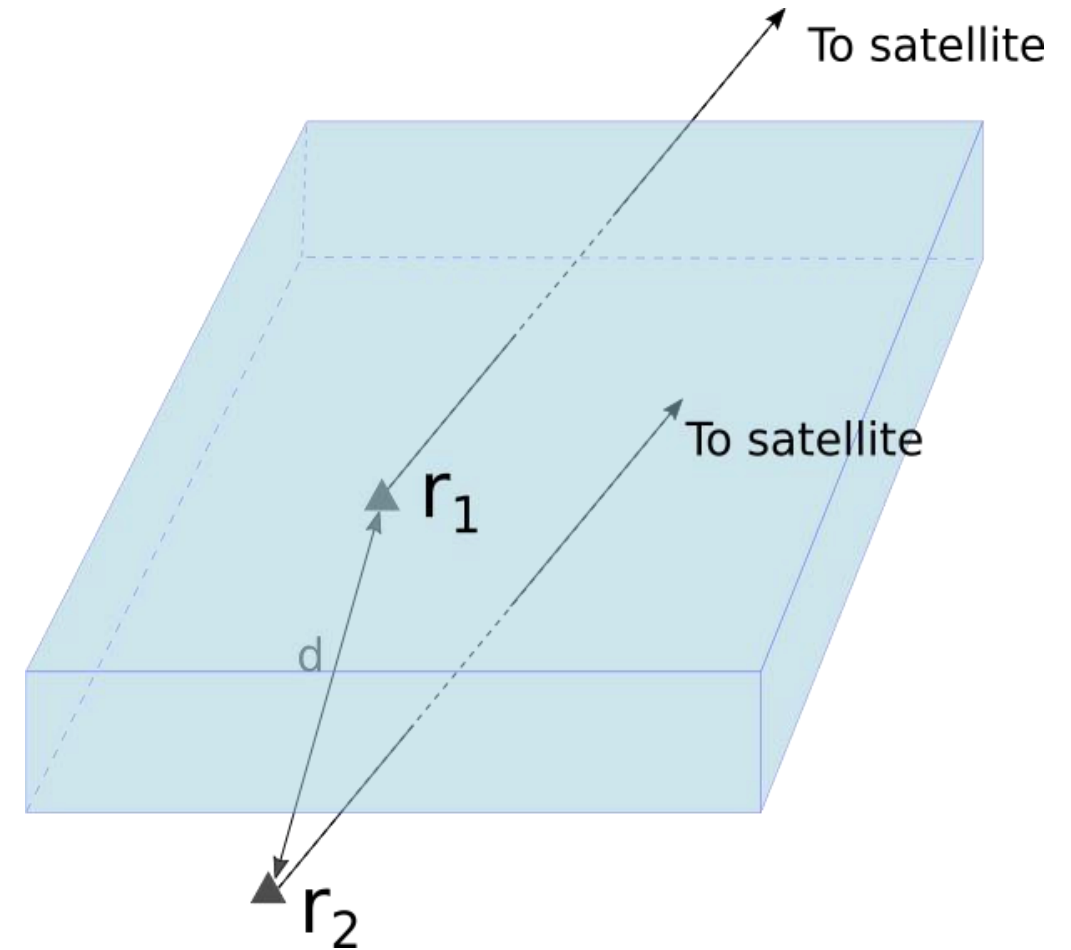
$$D(d) = \langle (\phi(x) - \phi(x + d))^2 \rangle = C * d^{5/3}$$

- If we assume that the variation in time are caused by spatial pattern moving in space (frozen flow) the same should be observed in time

$$D(t) = \langle (\phi(x) - \phi(x + t))^2 \rangle = C * t^{5/3}$$

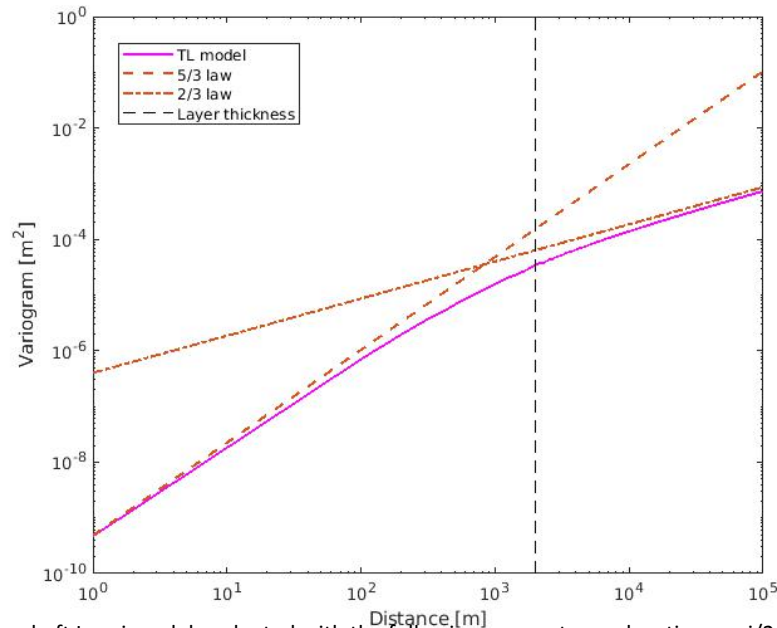
Treuhaft Lanyi Model (I)

- Describe the variation of the signal of two electromagnetic way passing trough a turbulent medium
- Assume a planar geometry and a layer with homogeneous isotropic turbulence.
- Gives the variogram of the signal given the distance of the receiver, and the azimuth and elevation of the satellite.
- Key element is the thickness of the turbulent layer

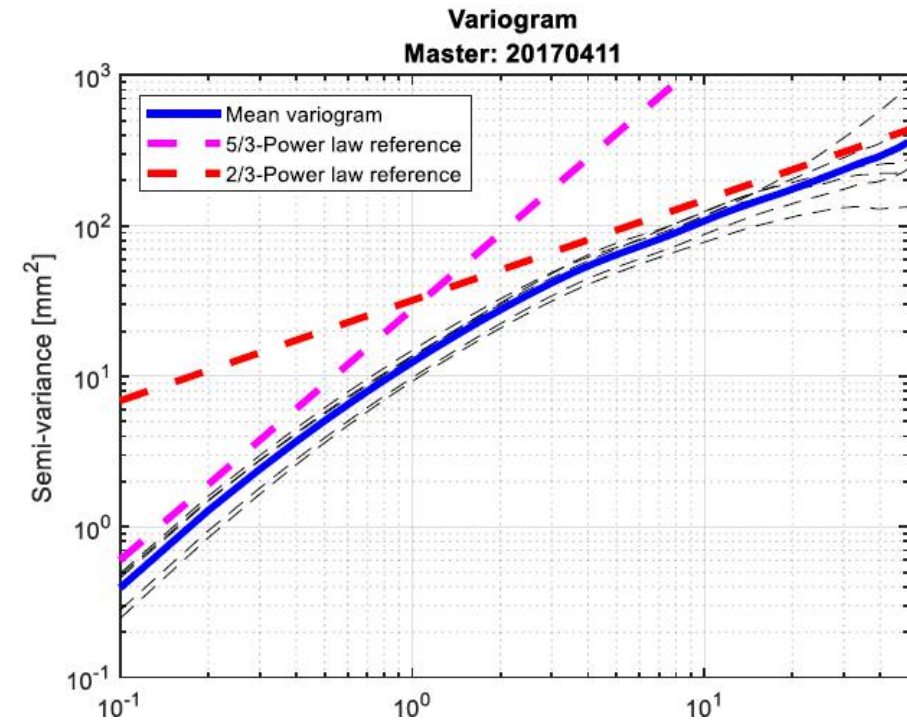


Treuhaft Lanyi Model (II)

- Two power law emerge form the model (in between smooth transition):
 - 5/3 when distance from station is much lower than the layer thickness
 - 2/3 when station distance is much larger that layer thickness
- Verified for the troposphere (VLBI, SAR, GNSS) observation
- Ionosphere is also a stratified medium so might be relevant



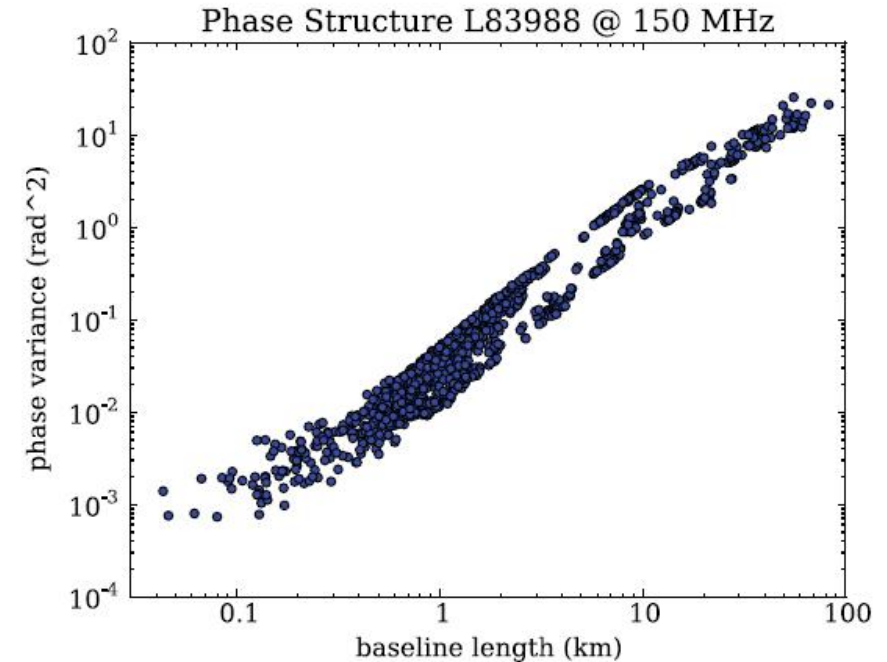
Treuhaft Lanyi model evaluated with the following parameters : elevation -> $\pi/2$ layer thickness -> 2 Km Turbulence constant -> $= 3 \cdot 10^{-7}$



Structure function of Atmospheric Phase Screen derived from Synthetic Aperture Radar (SAR) from Manzoni, Marco, et al. "Joint Exploitation of SAR and GNSS for Atmospheric Phase Screens Retrieval Aimed at Numerical Weather Prediction Model Ingestion." *Remote Sensing* 12.4 (2020): 654.

LOFAR Study

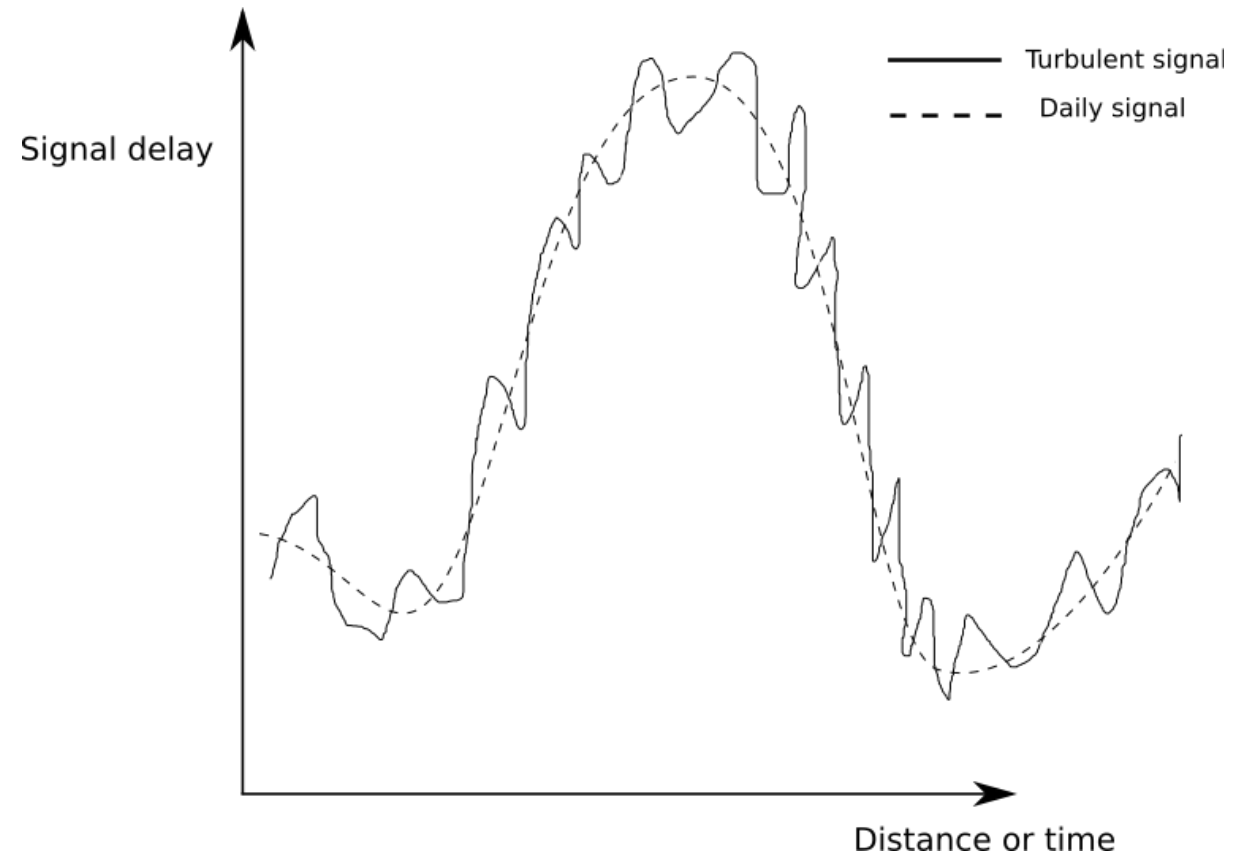
- Recently (Mevius et al 2016) structure function for the ionospheric delay have been evaluated using the LOW-Frequency Radio interferometer ARray (LOFAR)
- They found power law of ~ 1.9 greater than 1.66 (5/3) (Kolmogorov 3D range)



From :Mevius, M., et al. "Probing ionospheric structures using the LOFAR radio telescope." Radio Science 51.7 (2016): 927-941.

Sun Influence

- Sun is the main driver of ionospheric variation (daily variation)
- This should have an impact on the signal variogram
- I would make sense to remove such daily variation from the signal and test whether the residual has a turbulent behaviour



TEC Caused Delay on GNSS Signals (I)

- The GNSS signal is delayed by the Total Electron Component proportionally with the inverse of the frequency of the signal.
- GNSS satellites emit ranging signals on multiple frequencies this allows to make measurement of the integral of the TEC along the ray-path.
- At the main frequency L1 (GPS, GALILEO) 1 TEC $\rightarrow$ 16 cm of delay.
- Tracking of the phase of the signal can be done up to mm accuracy(~ 0.02 TEC) but with an unknown number of cycles.
- Differencing signal from same satellite/receiver and different frequencies all the geometric part can be removed (geometry free combination)

TEC Caused Delay on GNSS Signals (II)

$$\bullet \varphi_1 - \varphi_2 = \left(\frac{1}{f_1} - \frac{1}{f_2} \right) * \int TEC + N_1 \lambda_1 - N_2 \lambda_2 + \beta_1^r - \beta_1^r + \beta_1^s - \beta_1^s$$

Phase
difference

Terms depending on TEC
through frequency

Unknown number of cycle,
if signal tracking is not lost
constant in time

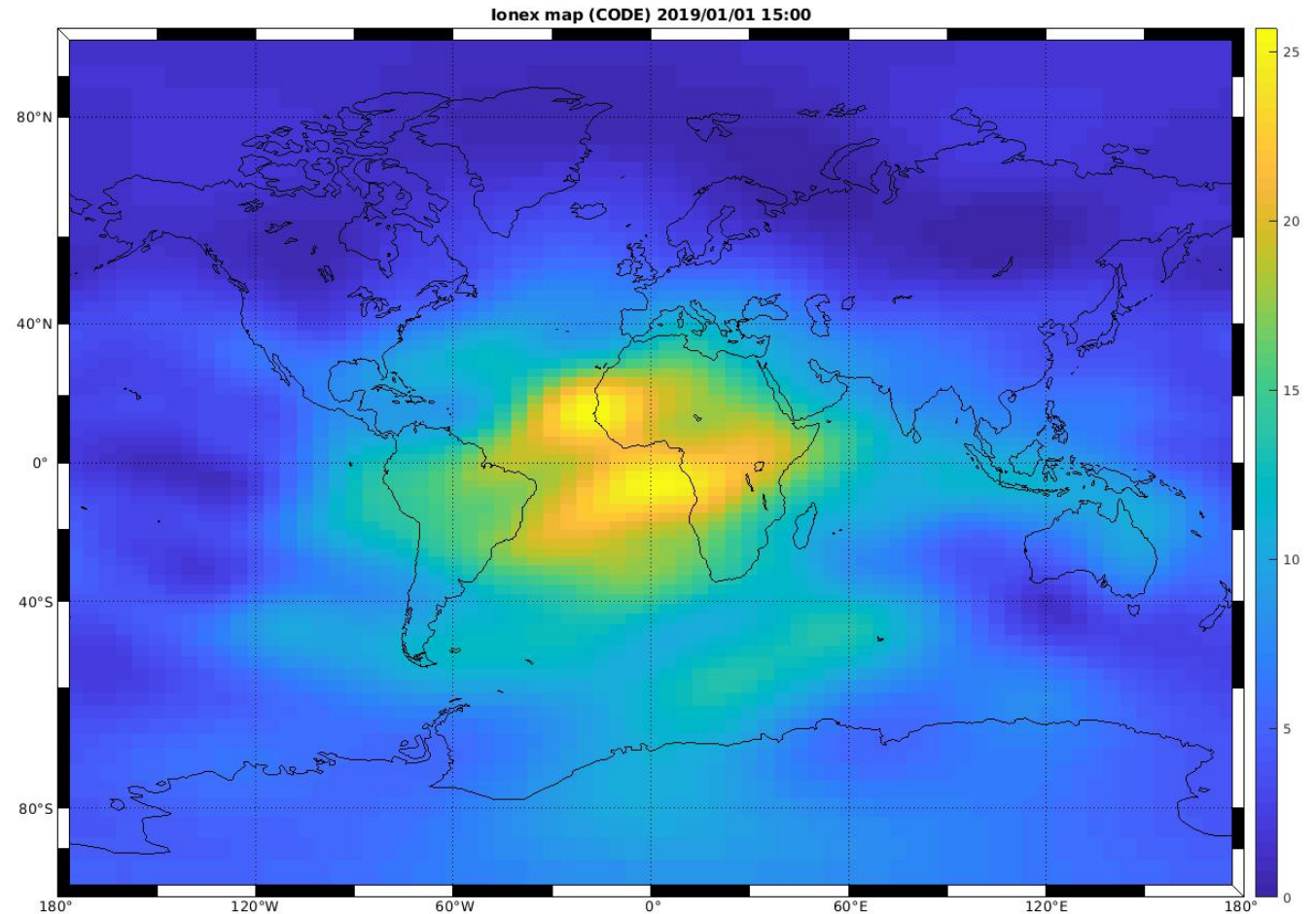
Electronic bias of the
receiver (mostly constant
in time) same for two
satellite seen from the
same receiver

Electronic bias of the
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- Differencing in time all the terms non depending from ionosphere disappears

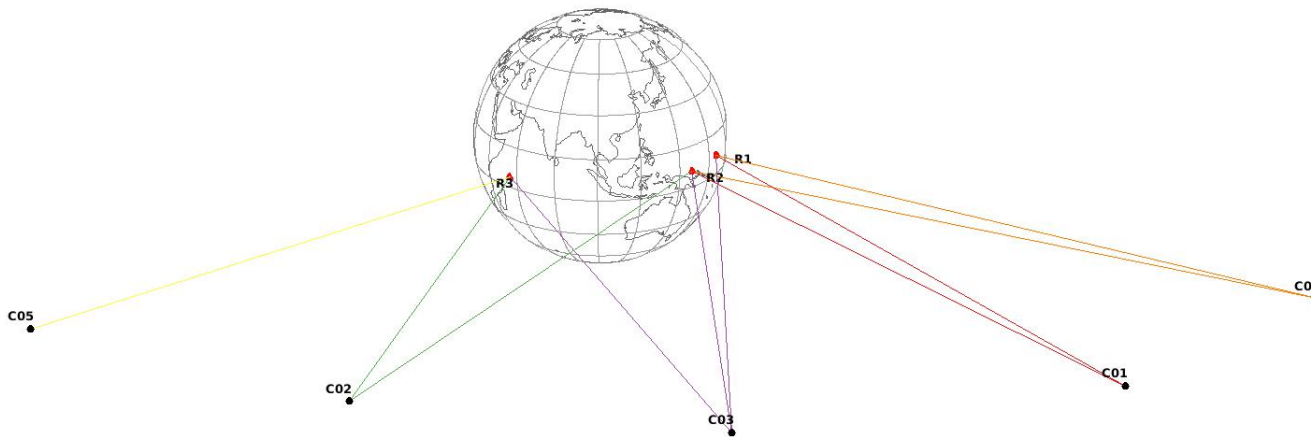
Daily Variation

- We need a model to remove daily variation
- IONEX model form IGS centers is a good candidate:
 - single layer model
 - 3D grid with resolution of 2.5° in latitude 5° in longitude and 1h in time

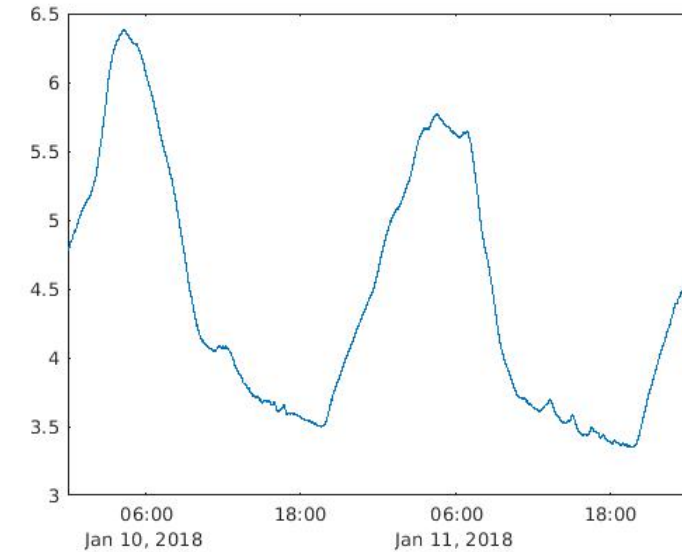


Geostationary Beidou (I)

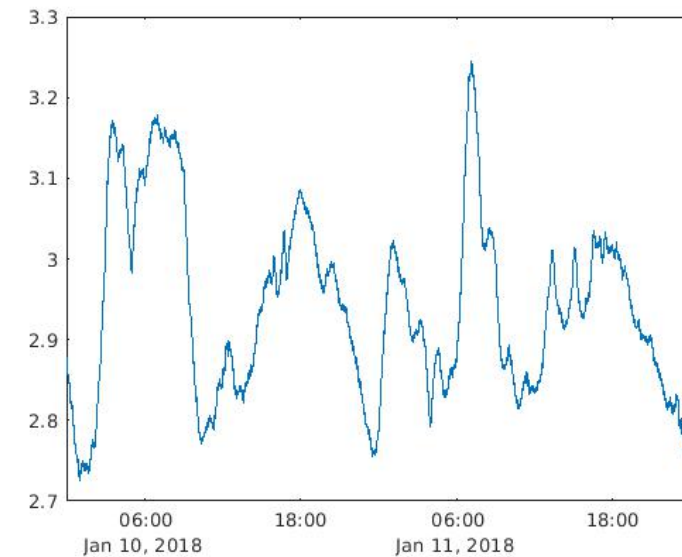
- Beidou Satellite System has 5 geostationary satellites.
- Their signal pass through the same ionosphere (almost).
- It is good chance to study the variability in time.
- 3 receivers studied along the equator (data from IGS network)



Geometry free Receiver 2 satellite C01 raw signal

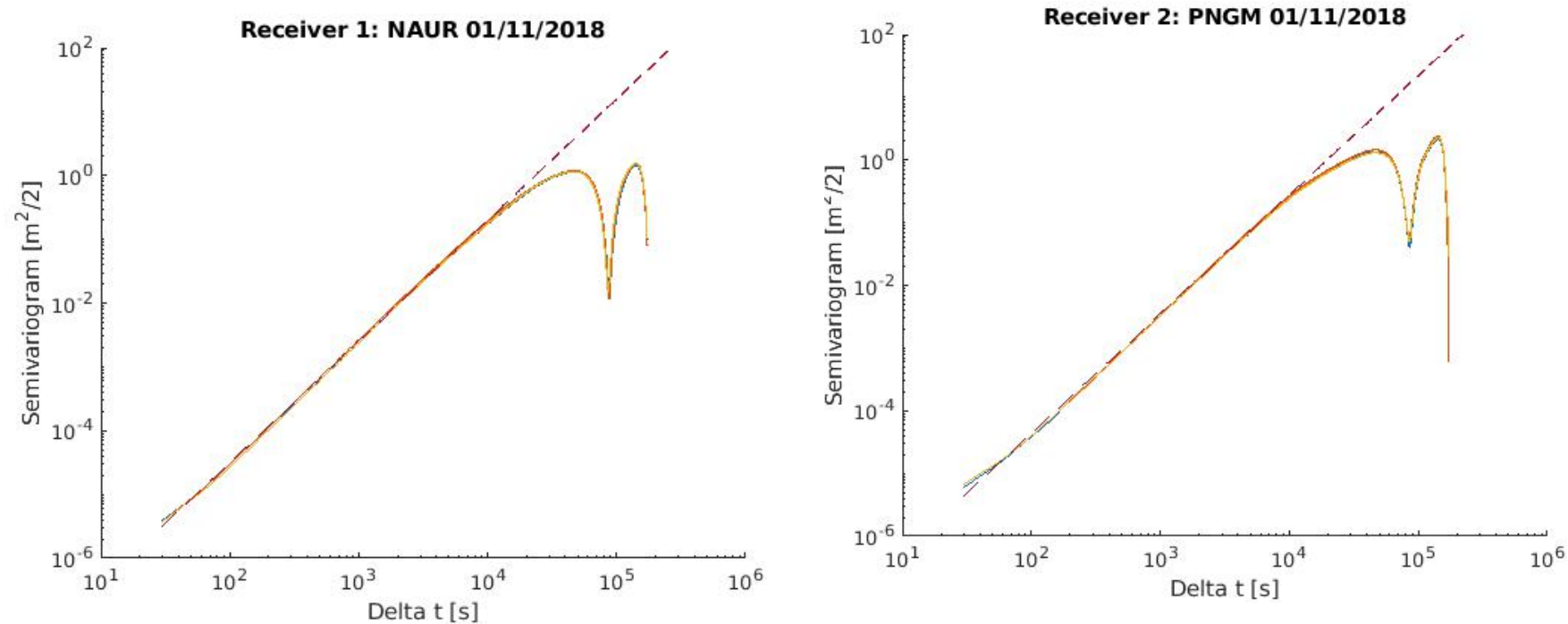


Geometry free Receiver 2 satellite C01 IONEX model removed



Geostationary Beidou (II)

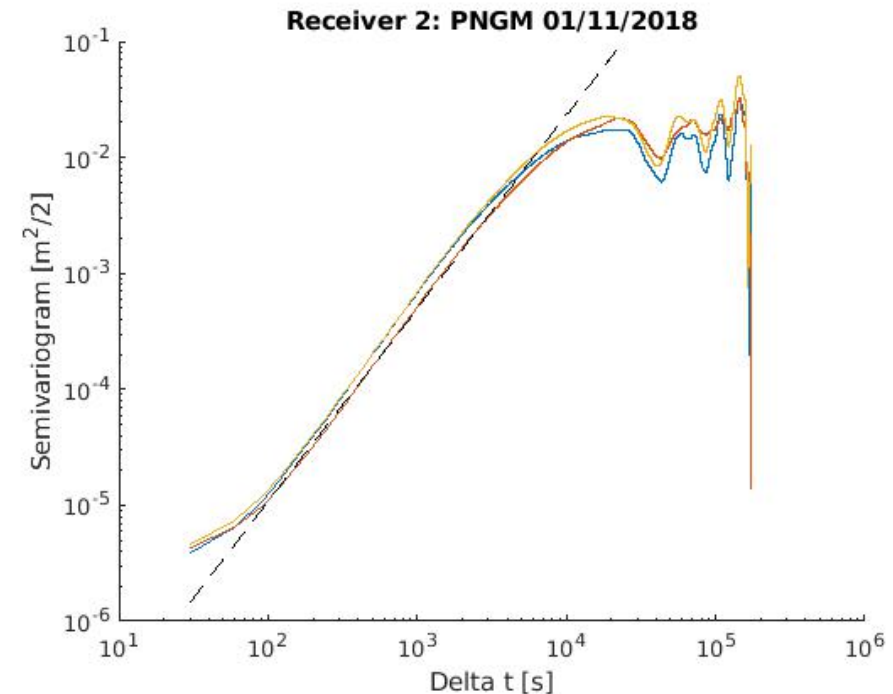
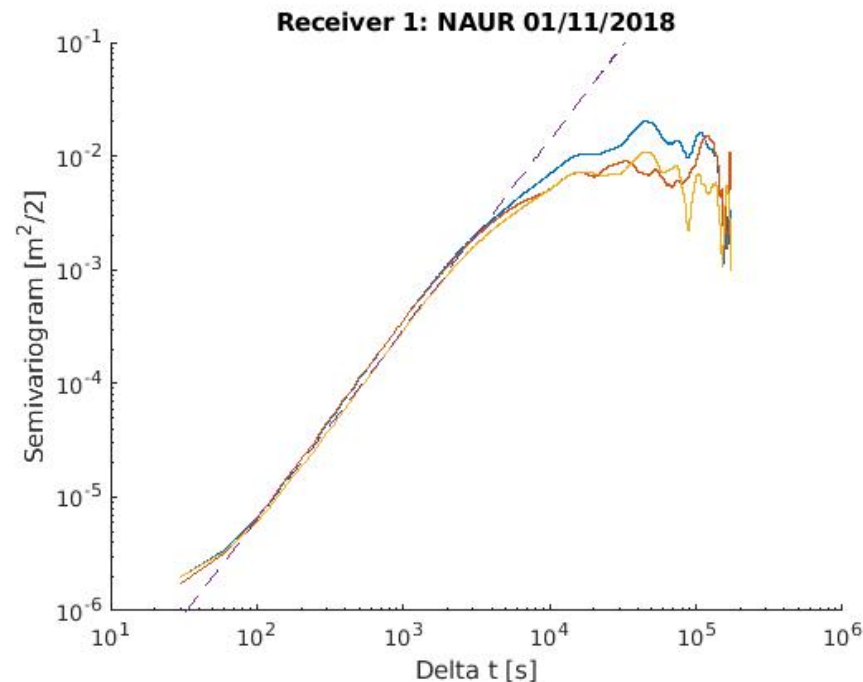
- 1.9 power law for the raw signal (same as LOFAR)



Structure function of the signal exhibiting a 1.9 power law (dashed line)

Geostationary Beidou (III)

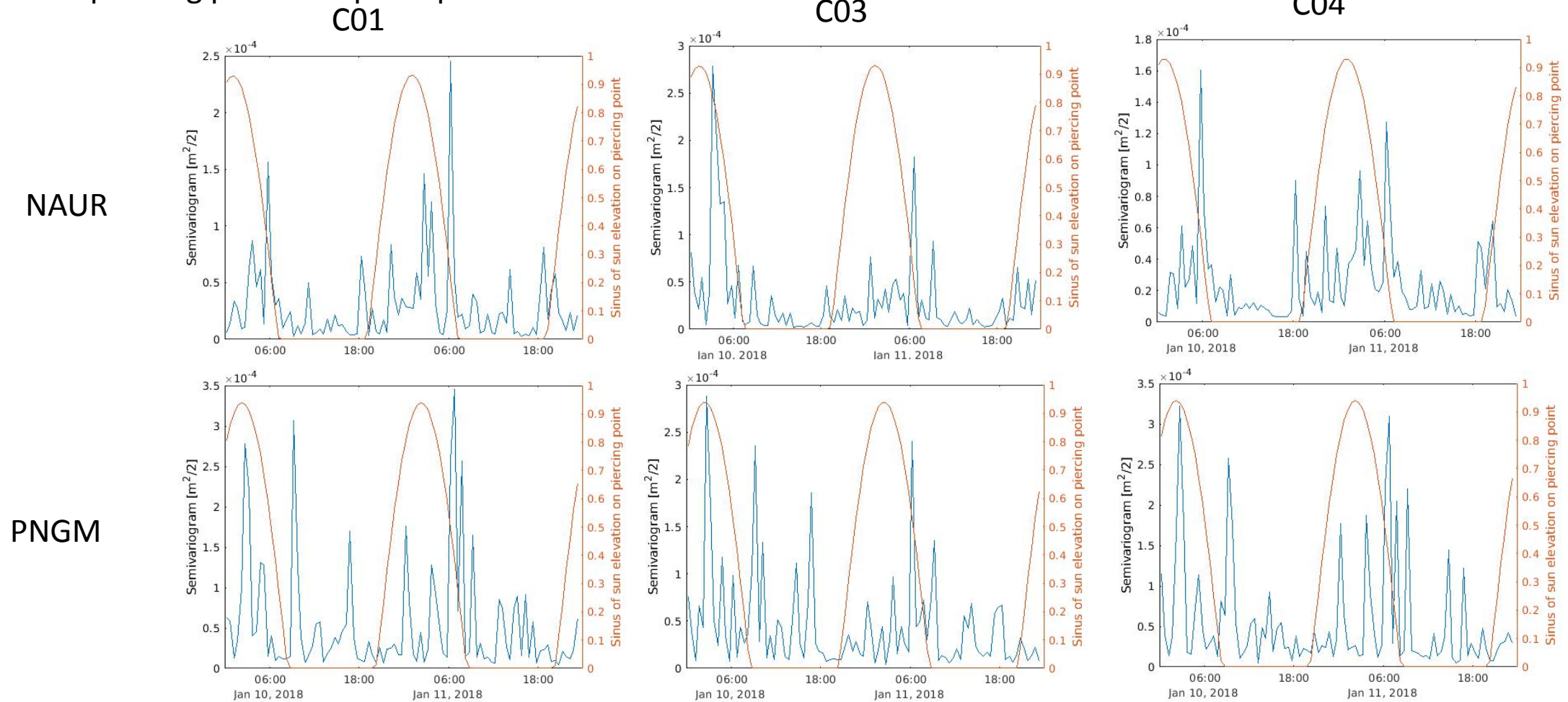
- Once the daily signal is removed the structure function exhibit a power law close to 5/3
- No clear sign of 2/3 power law



Structure function of the signal with 5/3 power law superimposed (dashed line)

Does the Variability Depend on Sun Flux?

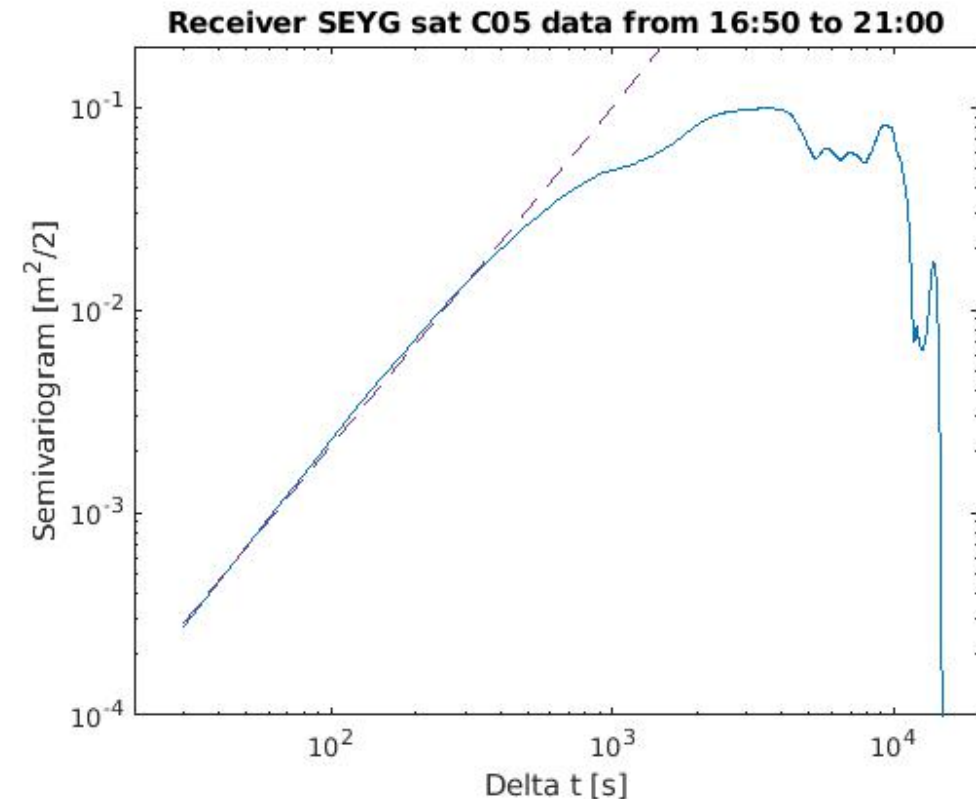
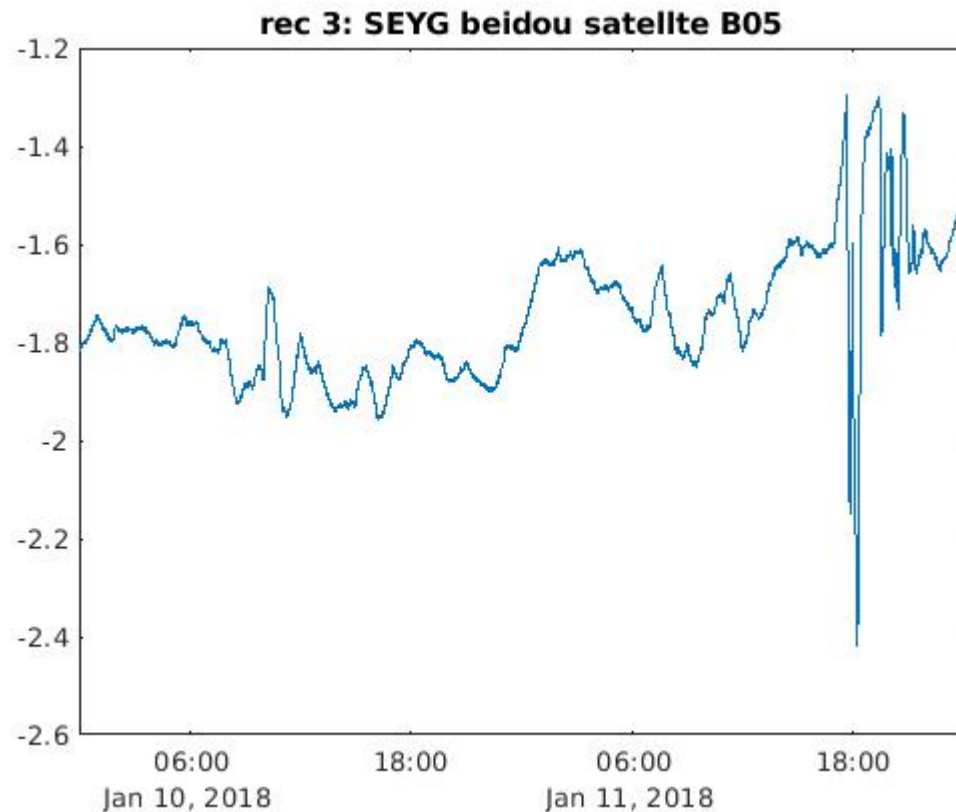
Variance at 240 second distance. The variance is computed each half hour. The sinus of sun elevation on the piercing point is superimposed



light dependence but not very clear

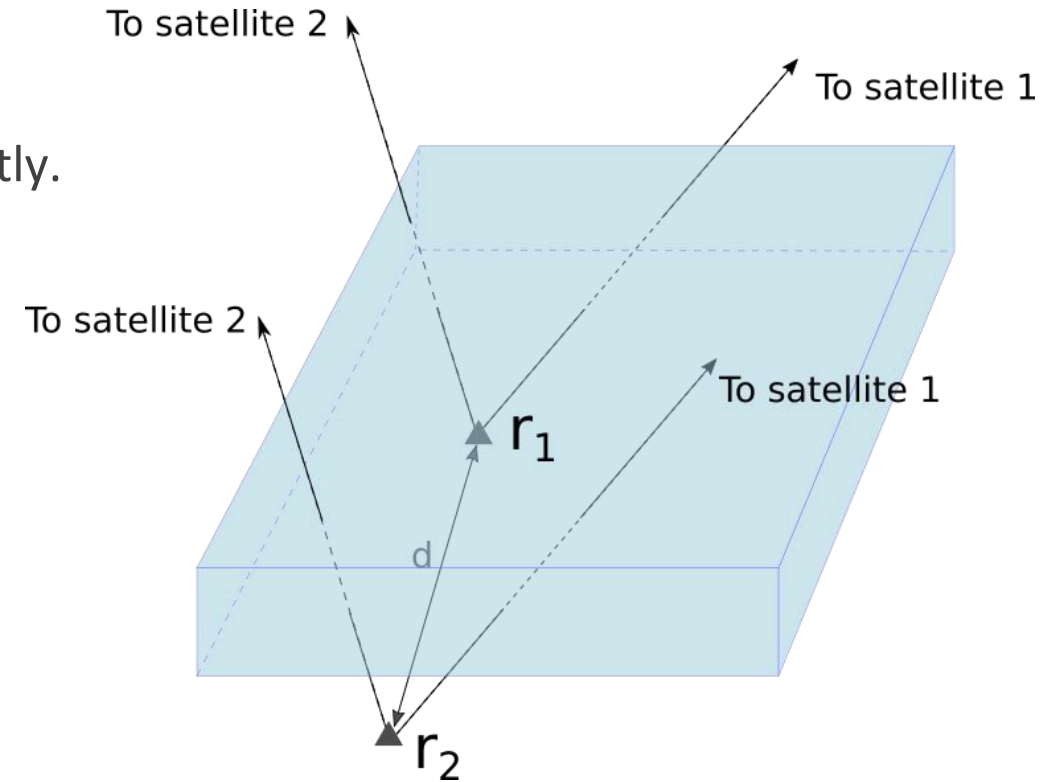
Special Case: Scintillation

- Scintillation was detected on receiver 3 data
- Interesting to study its variogram
- Also close to 5/3 power law



Spatial variogram : Double Difference Analysis

- Evaluate the variogram in space would be interesting.
- Ambiguity terms and electronic bias prevent this analysis directly.
- Ambiguity terms can be fixed (and thus eliminated) in post processing
- Electronic bias can be removed through double differencing of geometry free observables (gf).
- $\Delta \nabla gf = (gf_{r_1}^{s1} - gf_{r_2}^{s1}) - (gf_{r_1}^{s2} - gf_{r_2}^{s2})$
- Treuhaft Lanyi model is suitable only for single difference, it has to be extended to double difference (DD)



Treuhaft-Lanyi model for DD

- The Treuhaft-Lanyi model has been extended to Double difference DD, the variogram can be written as:

$$V(d, \theta_1, \theta_2, \phi_1, \phi_2, C_0) = C_0 \left(\frac{1}{\sin(\theta_1)^2} \int_{h_1}^{h_2} \int_{h_1}^{h_2} D(\mathbf{r}_1^1 - \mathbf{r}_2^1)^{2/3} - D(\mathbf{r}_1^1 - \mathbf{r}_1^1)^{2/3} dz' \right. \\ \left. dz'' + \frac{1}{\sin(\theta_2)^2} \int_{h_1}^{h_2} \int_{h_1}^{h_2} D(\mathbf{r}_1^2 - \mathbf{r}_2^2)^{2/3} - D(\mathbf{r}_1^2 - \mathbf{r}_1^2)^{2/3} dz' dz'' \right. \\ \left. + \frac{1}{\sin(\theta_1) \cdot \sin(\theta_2)} \int_{h_1}^{h_2} \int_{h_1}^{h_2} 2 \cdot D(\mathbf{r}_1^1 - \mathbf{r}_2^2)^{2/3} - D(\mathbf{r}_2^1 + \mathbf{d} - \mathbf{r}_1^2)^{2/3} - D(\mathbf{r}_1^1 + \mathbf{d} - \mathbf{r}_2^2)^{2/3} dz' dz'' \right)$$

where d : is the distance between the receiver

$\theta_1 \theta_2$: are the two elevation for the satellites

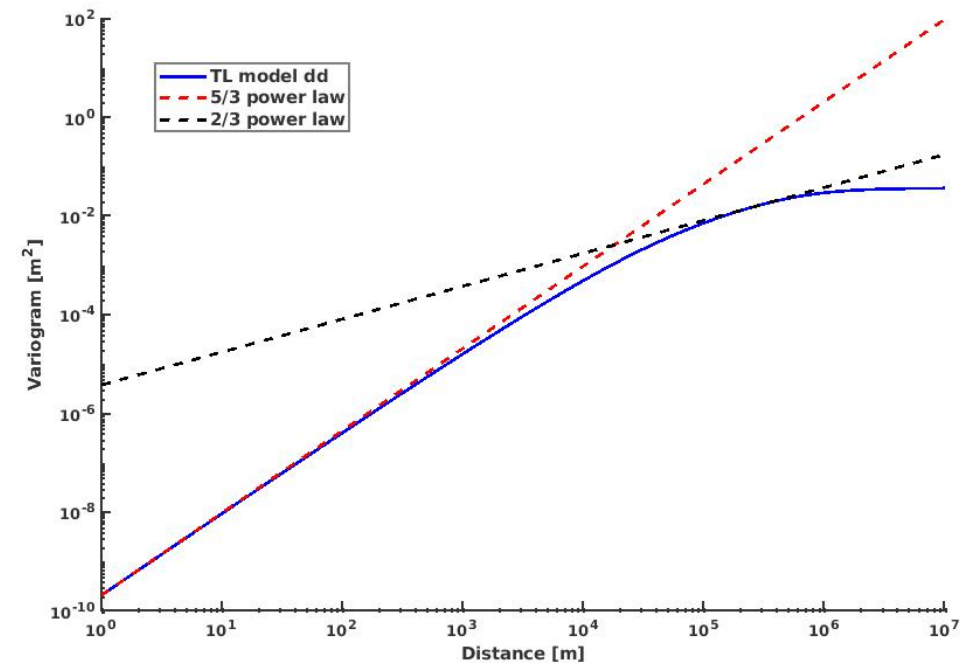
$\phi_1 \phi_2$: are the two elevation for the satellites

C_0 : is the turbulence constant

r_r^s : is the raypath from receiver r to satellite s along which the integration occurs

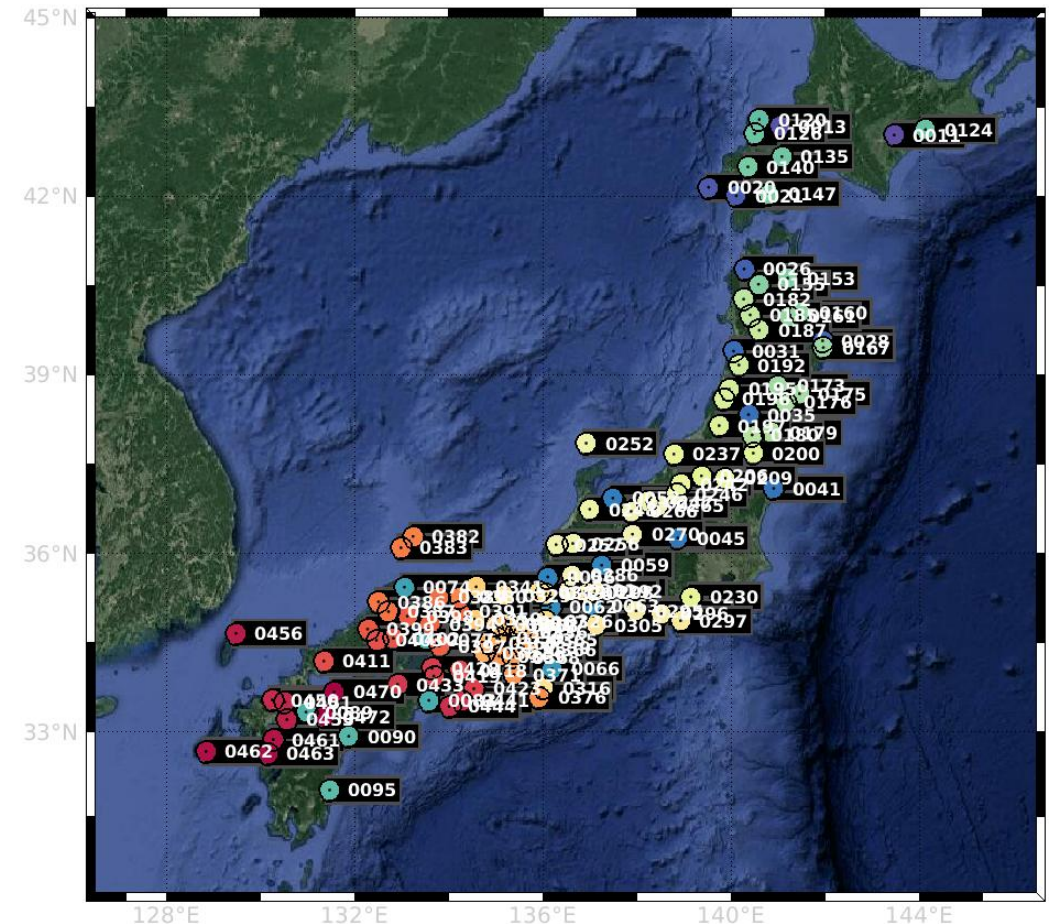
$D()$: is the euclidean distance operator

- The integrals can be evaluated numerically
- When using satellite using same elevation for both satellite the variogram saturates as in figure on the right.

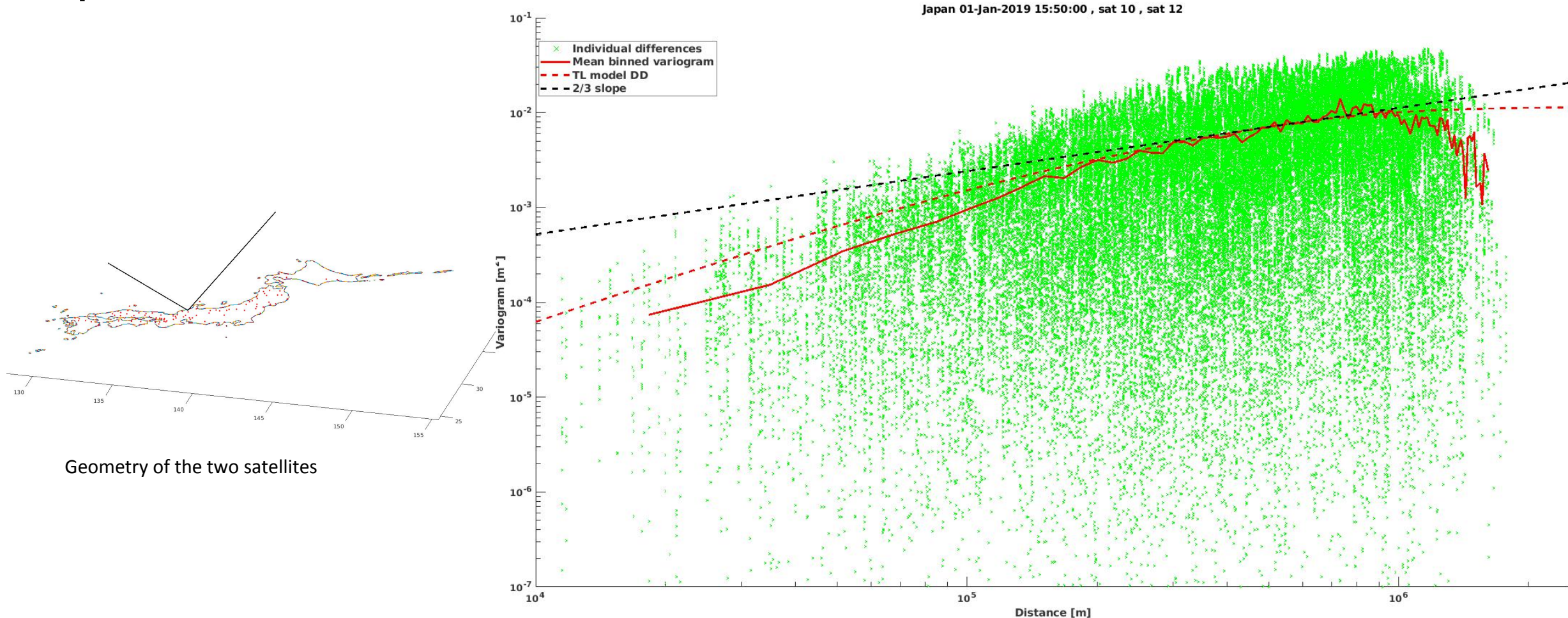


Case Study: Japan

- Japan has a dense network of homogeneous GNSS receiver (better for GPS ambiguity resolution) with precise GPS pseudorange code (needed to solve ambiguity in dual frequency case)
- Analysis on GPS L1 L2 frequencies.
- 120 station processed in network mode.



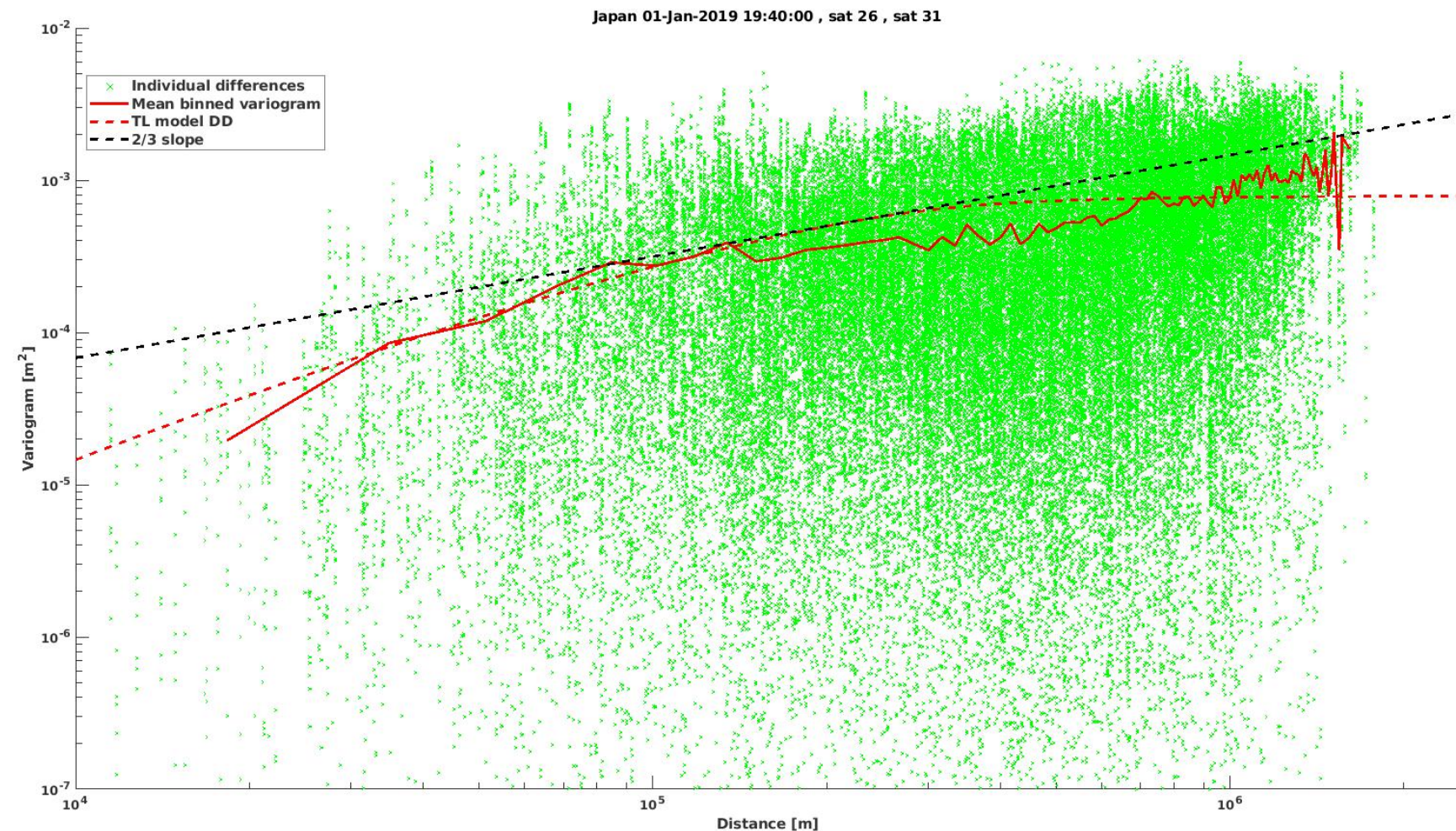
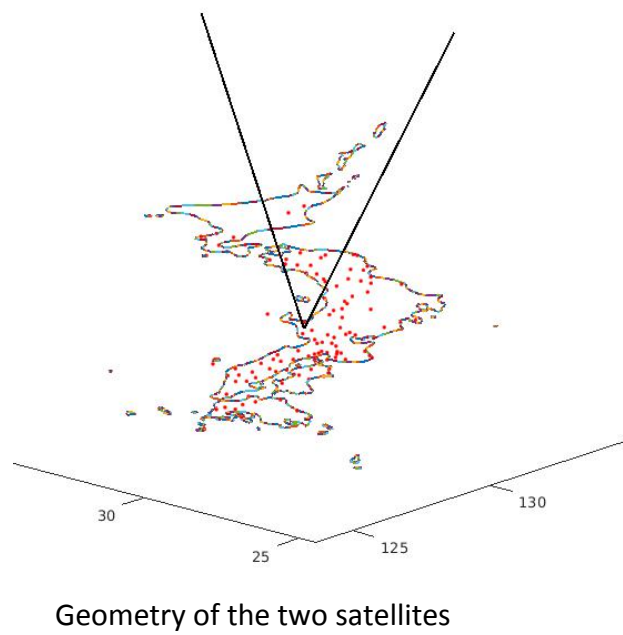
Spatial Structure Function (I)



Light agreement of model and data.

El 1 [°]	El 2 [°]	Az 1 [°]	Az 2 [°]	Layer height [km]	Layer thickness[km]	C0
38.03	40.56	-144.71	45.95	350	100	5.50E-09

Spatial Structure Function (II)



Light agreement of model and data.

El 1 [°]	El 2 [°]	Az 1 [°]	Az 2 [°]	Layer height [km]	Layer thickness[km]	C0
66.48	60.11	-2.62	113.19	350	100	3.0E-09

Conclusion

- The temporal and spatial variability of ionospheric delay on GNSS has been studied looking for Kolmogorov like turbulent behaviour
- Raw signal temporal structure function match the power law found in LOFAR studies
- A daily variation + turbulent signal model has been proposed
- Structure function in time computed using geostationary satellite show a power law close to the one from Kolmogorov turbulence once the daily signal is removed
- Treuhaft Lanyi model has been extended to double difference to study the spatial structure function from GNSS
- Experimental data computed from 1 day on dense GNSS Japanese network shows light agreement with the model

Future work

- Need for denser data in space -> PPP processing with ambiguity resolution
- Calibrate Ionospheric bias of the receiver -> from double difference processing back to single difference processing
- The model has to be validated in more places on the earth
- Account for inhomogeneity in the vertical profile of turbulence