Continuum modelling of grain-size segregation in bedload transport

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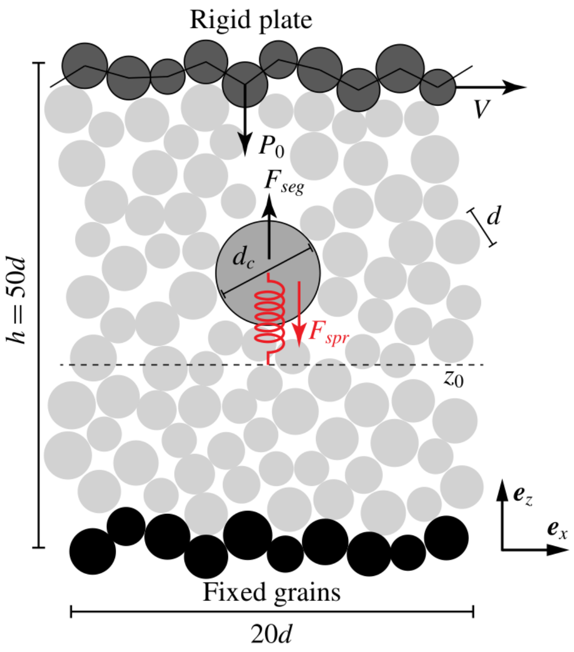
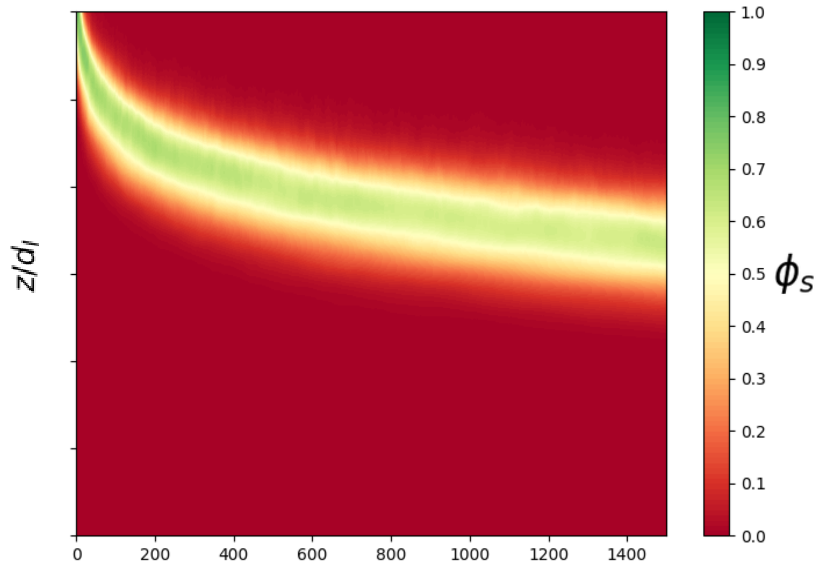
Objective of this work

Bridge the gap between



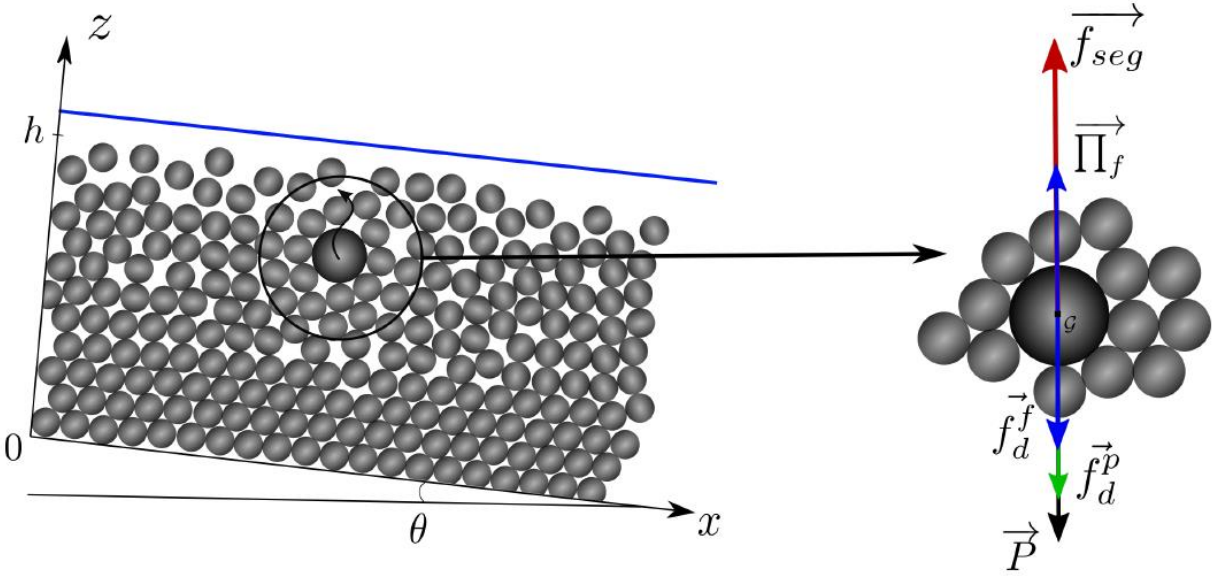
Particle scale forces for size segregation (Guillard et al. 2016)

?



Continuum modelling of size segregation

Segregation forces on a single particle in a bath of small particles

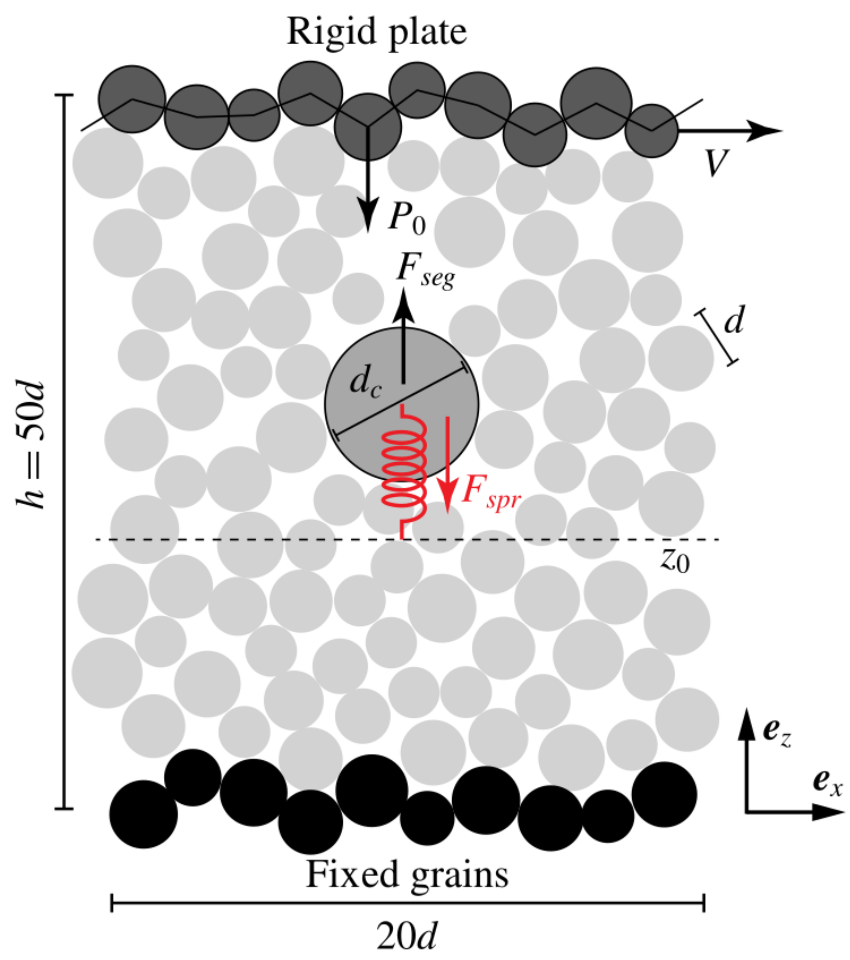


Vertical Lagrangian equation of the large intruder: $$\rho^p V_l \frac{dw^l}{dt} = P - \Pi_f + f_d^f + f_d^p - f_{seg}$$

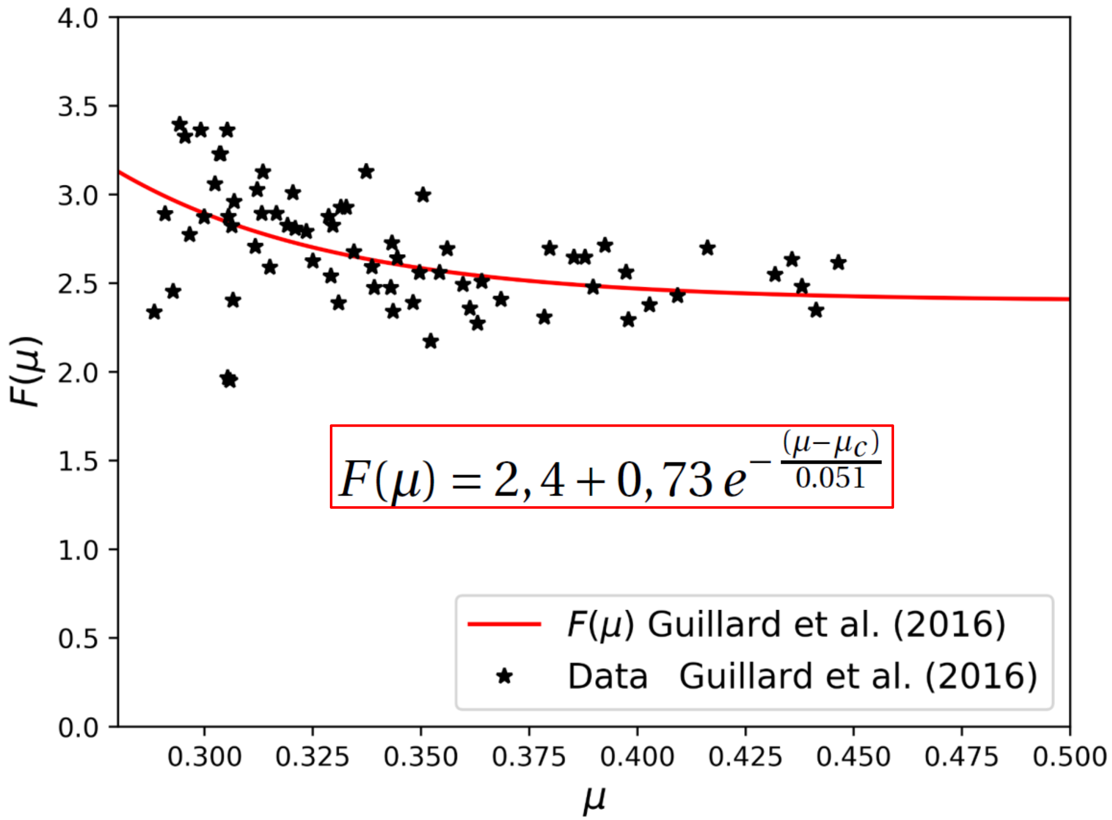
- Solid drag force: $f_{seg} = V_l \underline{\mathcal{F}(\mu)} \dfrac{\partial P^s}{\partial z}$ Tripathi and Khakhar (2011) $\sim$ Stokesian drag force
- Segregation force: $f_d^p = c\pi\eta^p d_l \left(w^s - w^l\right)$ Guillard et al. (2016) $\sim$ Buoyancy force

How to express F(μ) ?

2D DEM Simulations:

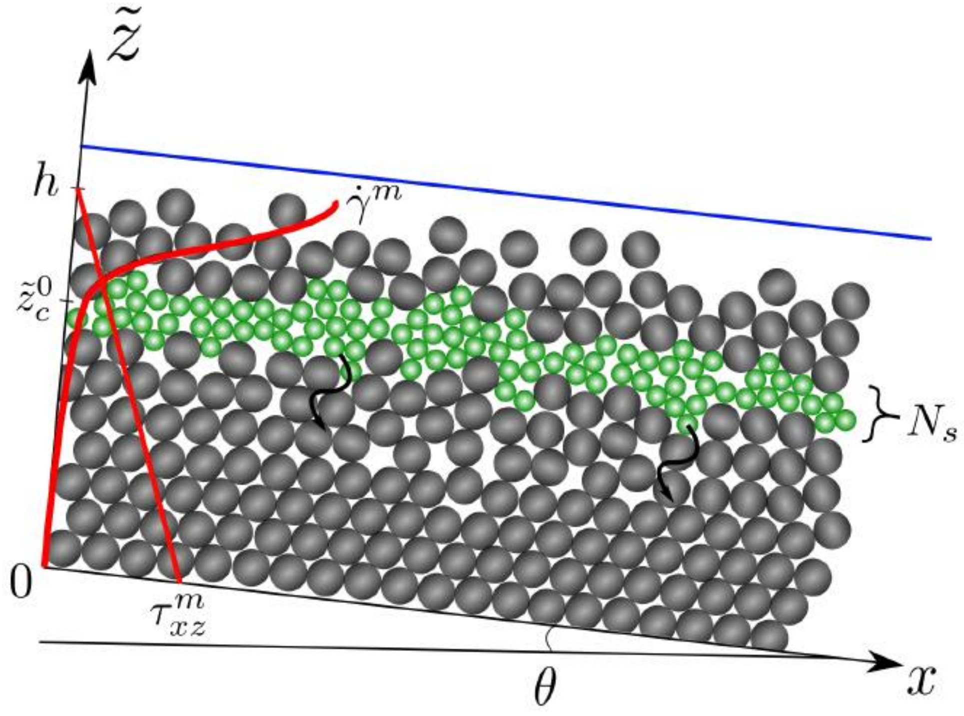


Segregation force: $$f_{seg} = V_l \mathcal{F}(\mu) \frac{\partial P^s}{\partial z}$$



From Guillard et al. (2016)

Upscaling to numerous particles…necessity of a continuum model



3 continuum phases:

- Fluid
- Large particles
- Small particles

Multi-phase flow model equations

Number of large particle per unit volume

Fluid drag force

Fluid momentum balance:

$$\rho^f \left[\frac{\partial \epsilon w^f}{\partial t} + \frac{\partial \epsilon w^f w^f}{\partial z} \right] = -\epsilon \frac{\partial p^f}{\partial z} - \rho^f g \cos\theta - \boxed{n_l < f_d^{f\to l} >} - \boxed{n_s < f_d^{f\to s} >}$$

Small particles momentum balance:

$$\rho^p \left[\frac{\partial \Phi^s w^s}{\partial t} + \frac{\partial \Phi^s w^s w^s}{\partial z} \right] = -\frac{\partial p^s}{\partial z} - \Phi^s \frac{\partial p^f}{\partial z} - \rho^p g \cos\theta + \boxed{n_s < f_d^{f\to s} >} + n_s < f_{l\to s} >$$

Large particles momentum balance:

$$\rho^p \left[\frac{\partial \Phi^l w^l}{\partial t} + \frac{\partial \Phi^l w^l w^l}{\partial z} \right] = -\frac{\partial p^l}{\partial z} - \Phi^l \frac{\partial p^f}{\partial z} - \rho^p g \cos\theta + \boxed{n_l < f_d^{f\to l} >} + \boxed{n_l < f_{s\to l} >}$$

$$n_l < f_{s\to l} > = \frac{\rho^p \Phi^l}{t_{ls}} (w^s - w^l) + \Phi^l \mathcal{F}(\mu) \frac{\partial p^m}{\partial z}$$

Derivation of the advection-diffusion equation

The small particles momentum balance is made dimensionless:

$$\cancel{\frac{\partial \phi^s \tilde{w}^s}{\partial \tilde{t}}} + \cancel{\frac{\partial \phi^s \tilde{w}^s \tilde{w}^s}{\partial \tilde{z}}} = -\frac{\tilde{p}^m}{\Phi}\frac{\partial \phi^s}{\partial \tilde{z}} + \frac{\phi^s}{St^f}\cancel{\left(\tilde{w}^f - \tilde{w}^s\right)} - \frac{\left(\tilde{w}^s - \tilde{w}^m\right)}{St^p} + \phi^l \mathcal{F}(\mu)\frac{\partial \tilde{p}^m}{\partial \tilde{z}}$$

with $$St^p = \frac{\rho^p d_l W}{6c\eta^p}$$

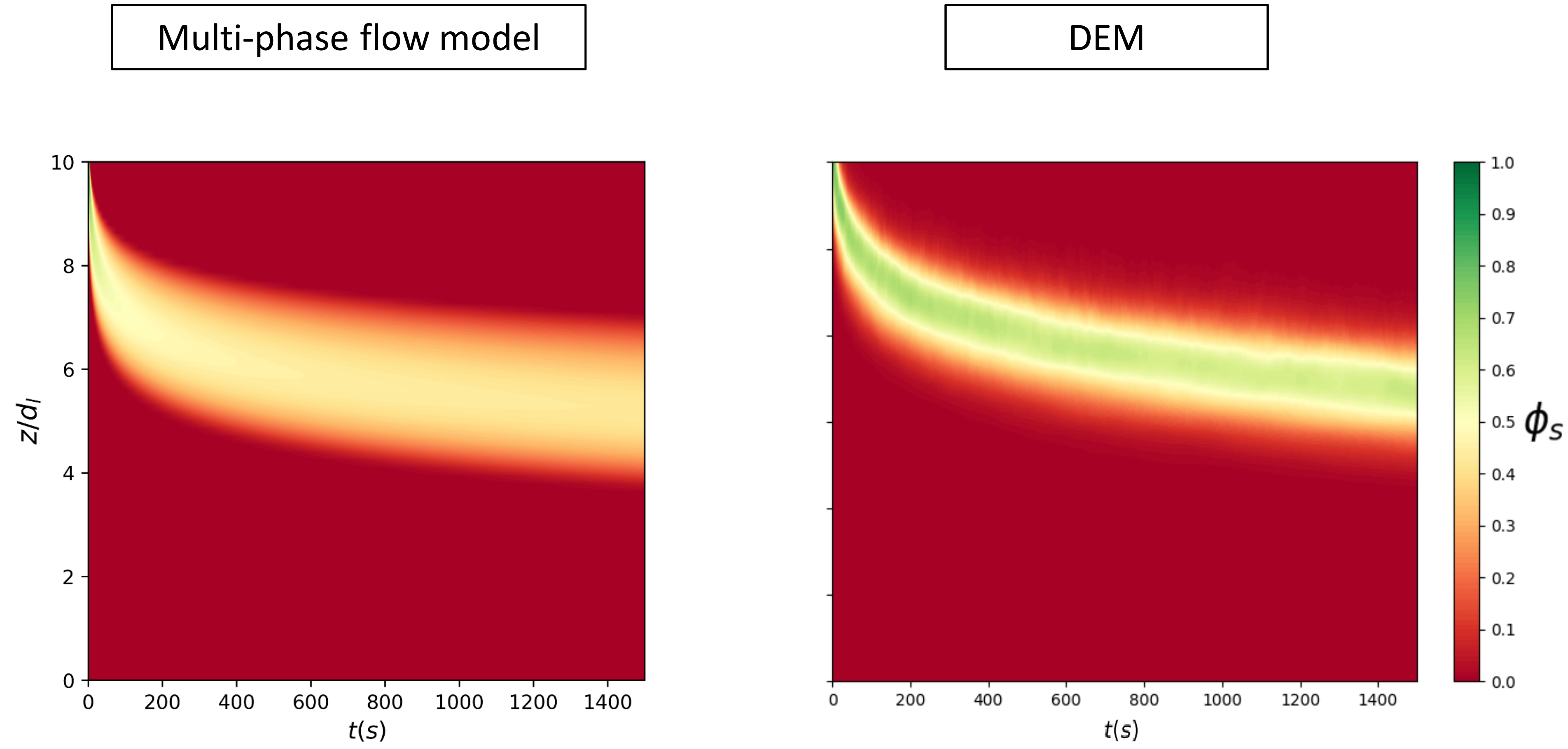
$$\phi^s \tilde{w}^s = -\frac{\phi^s}{\Phi}\tilde{p}^m St^p \frac{\partial \phi^s}{\partial \tilde{z}} + \phi^l \phi^s \mathcal{F}(\mu) St^p \frac{\partial \tilde{p}^m}{\partial \tilde{z}}$$

$$\frac{\partial \phi^s}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{z}}\left(\phi^l \phi^s S_r\right) = \frac{\partial}{\partial \tilde{z}}\left(D\frac{\partial \phi^s}{\partial \tilde{z}}\right)$$

$$S_r = \mathcal{F}(\mu) St^p \frac{\partial \tilde{p}^m}{\partial \tilde{z}}$$

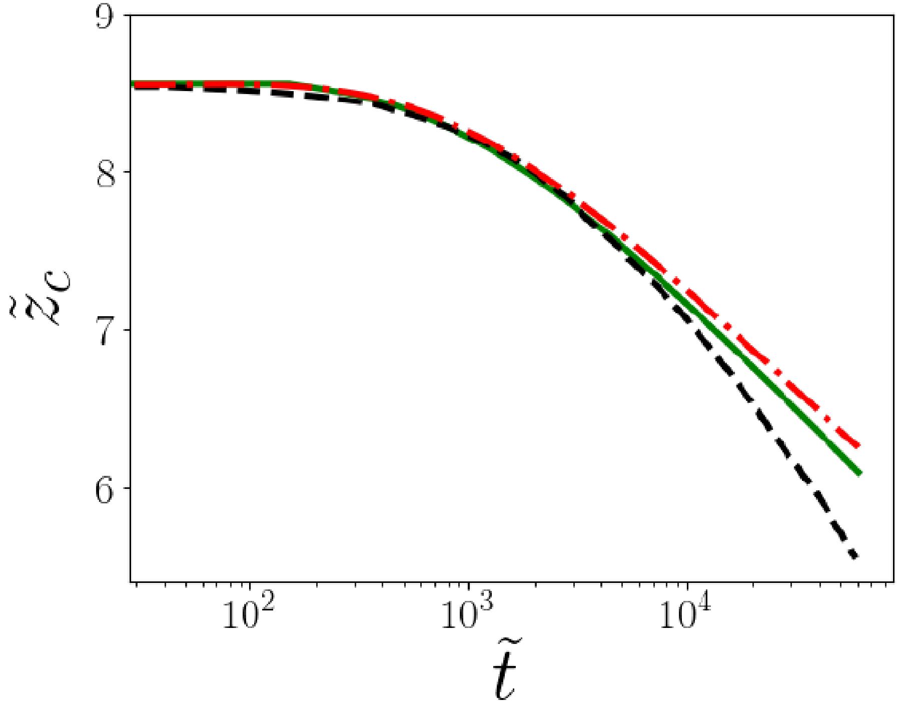
$$D = \frac{\phi^s \tilde{p}^m St^p}{\Phi}$$

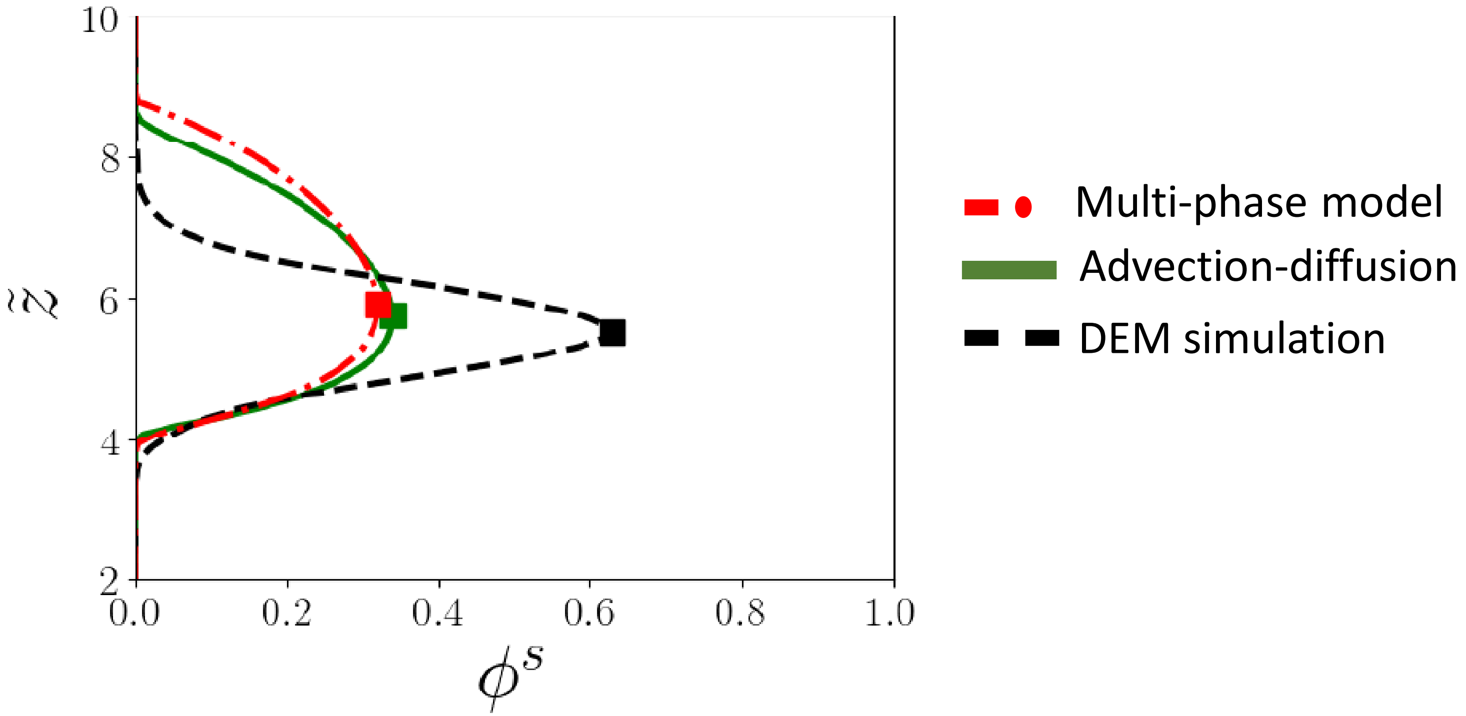
Results against DEM simulations of Chassagne et al. 2020



- Small particle dynamics is qualitatively reproduced
- Too much diffusion of small particles concentration

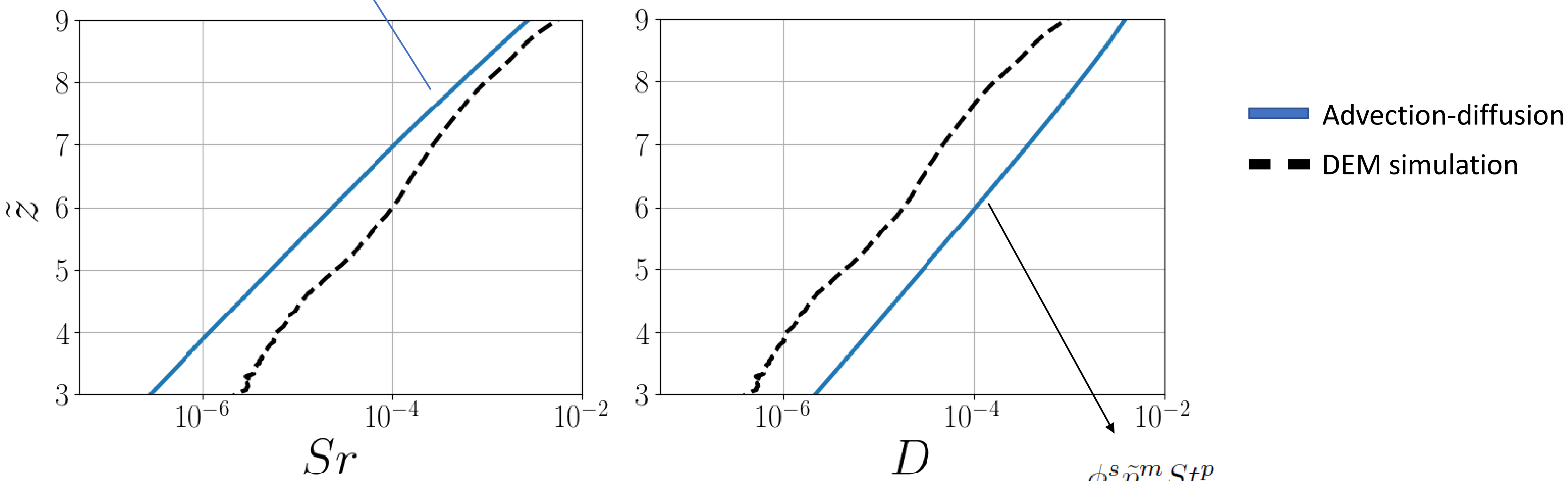
Comparison between the multi-phase flow model and the advection-diffusion equation





Comparison of the coefficients with the DEM

$$S_r = \mathcal{F}(\mu) St^p \frac{\partial \tilde{p}^m}{\partial \tilde{z}}$$



$$D = \frac{\phi^s \tilde{p}^m St^p}{\Phi}$$