

## Abstract

The main mechanism of transport of magma in the Earth's crust is the formation of cracks, or dikes, through which the melt moves toward the surface under the action of buoyancy forces and tectonic stresses. Due to the structural features of the crust or external stress fields, dikes often do not reach the surface, but penetrate the localized region in which the rocks melt, leading to the formation of magmatic chambers, whose volume can exceed thousands of cubic kilometers.

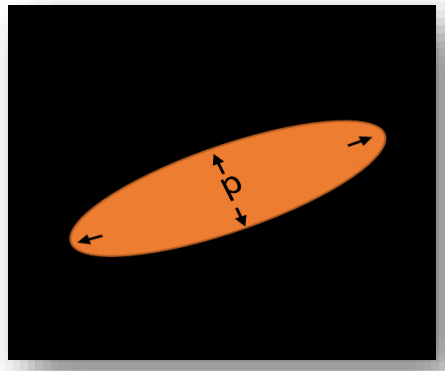
We present a model of the process of formation of a magma chamber during the intrusion of dikes with a given flow rate. The model, which is based on the solution of heat equation, considers the actual melting diagrams of magma and rocks. It is shown that, in case of magmatic fluxes typical of island arc volcanoes, magma chambers are formed over hundreds of years from the beginning of magma intrusion. The influence of the magma flow rate, the size of the dikes and their orientation on the volume of the formed magma chamber and its shape was investigated. The size of the chamber significantly exceeds the area of dike intrusion due to the displacement of magma and rocks of the crust, their heating up and melting.

To calculate displacement of rock and magma in a numerical simulation, a hybrid method based on PIC/FLIP interpolation is developed, making it possible to avoid unphysical mixing due to numerical dissipation, thus preserving the fine details of the formed magma chamber.

## Mathematical model

Injection of individual dike leads to displacement of elastic host rocks, heat transfer, rock melting and magma solidification.

We model individual dike as an ellipsoid with semi-axes  $a$  and  $b$  and use analytical solution (Muskhelishvili, 2013) in order to calculate host rock displacement. Volume of the individual dike and frequency of emplacement is controlled by the specified feeding rate ( $\text{km}^3/\text{y}$ ). The model is 2D and the third dimension is specified and constant.



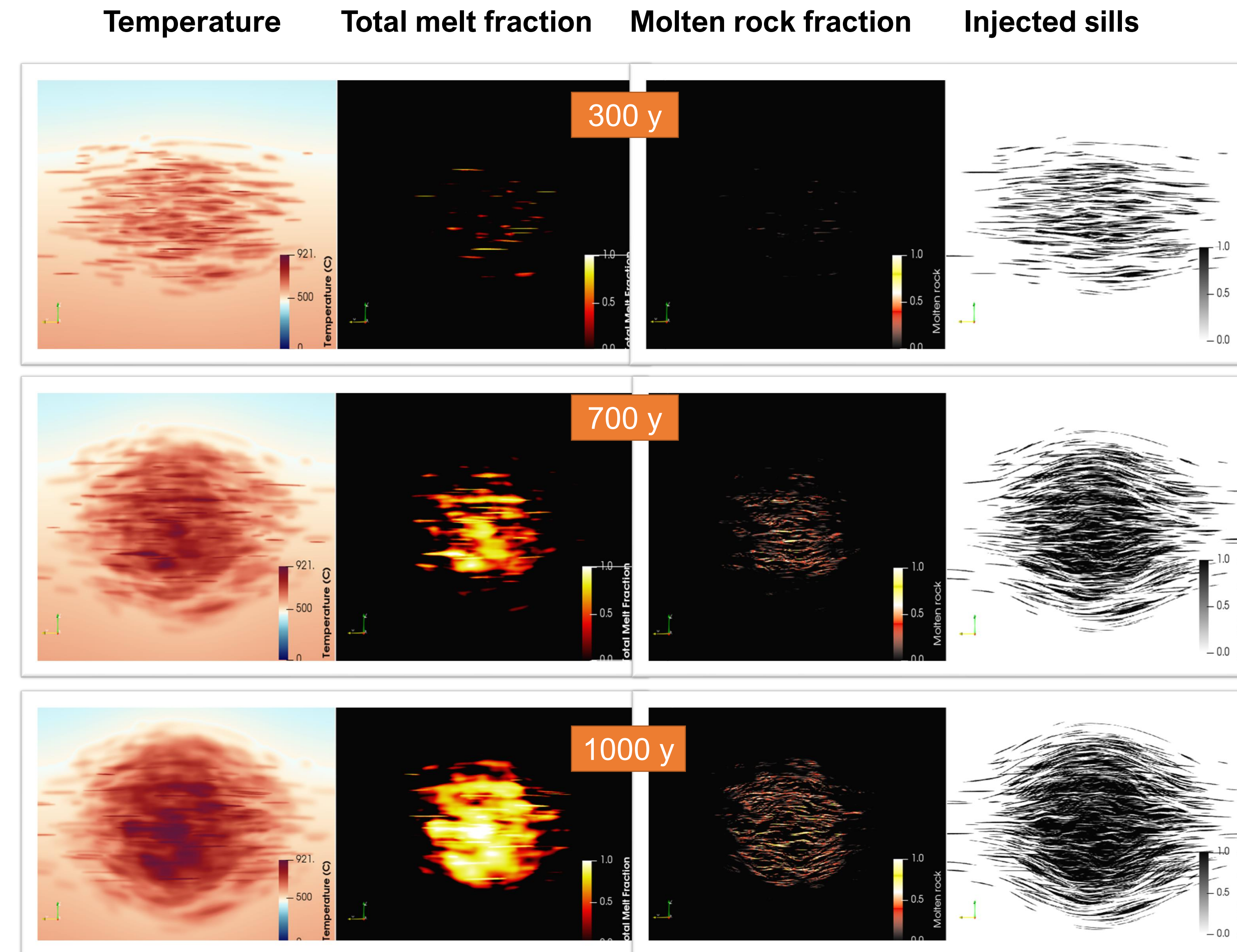
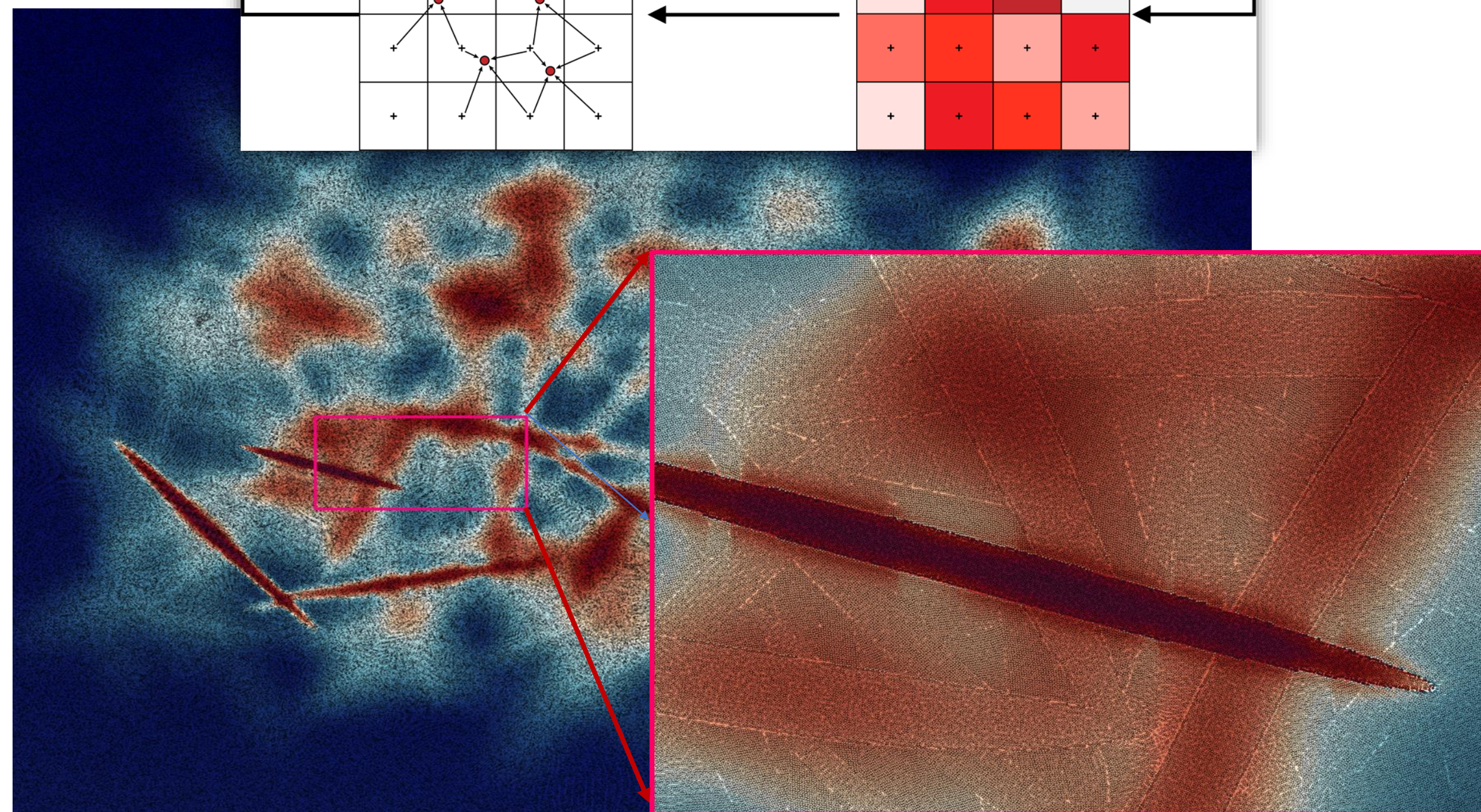
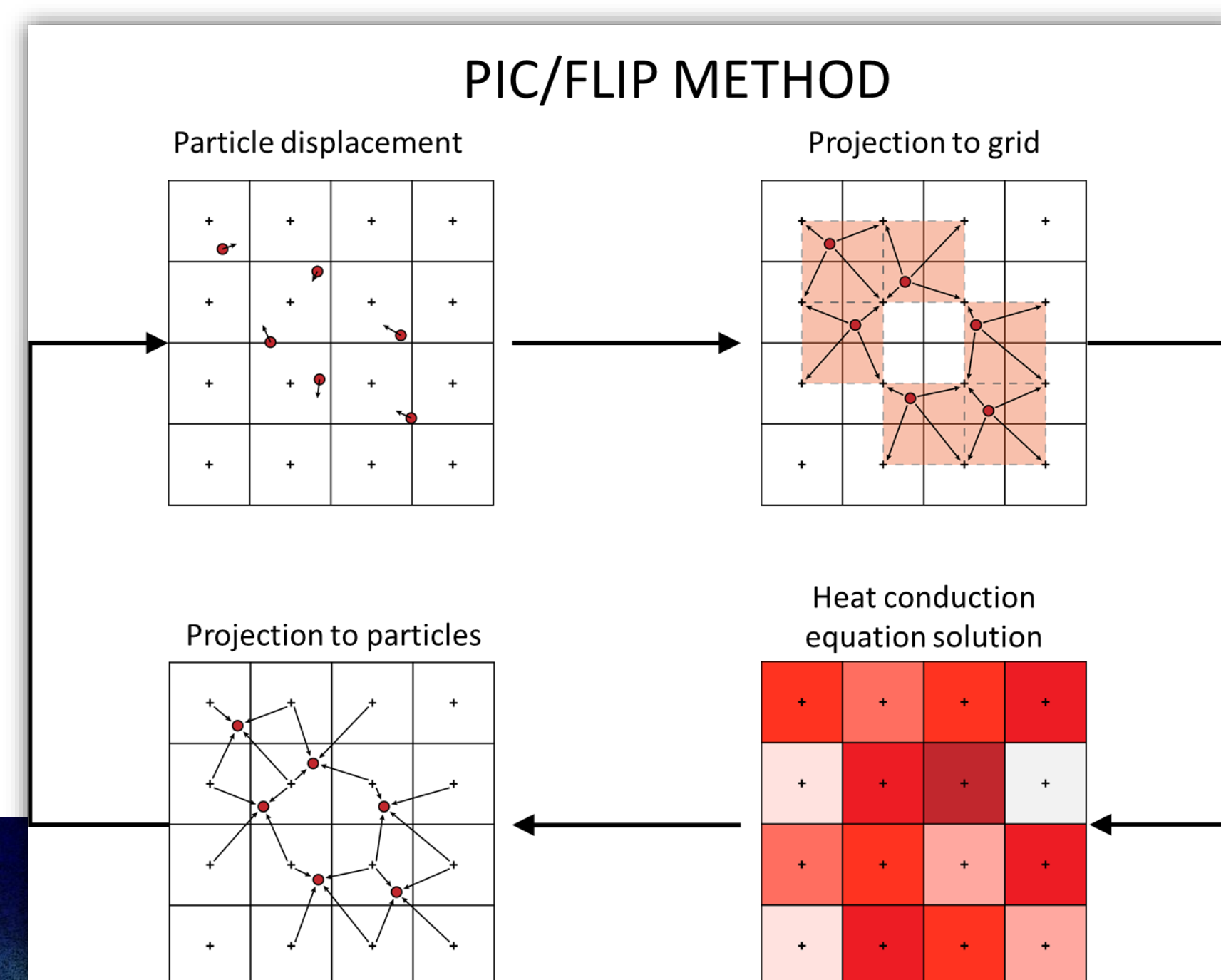
$$\rho C_p \left( \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{v} T \right) + \rho L \frac{\partial X}{\partial t} = \nabla \cdot \mathbf{k} \nabla T$$

$X$  – melt fraction

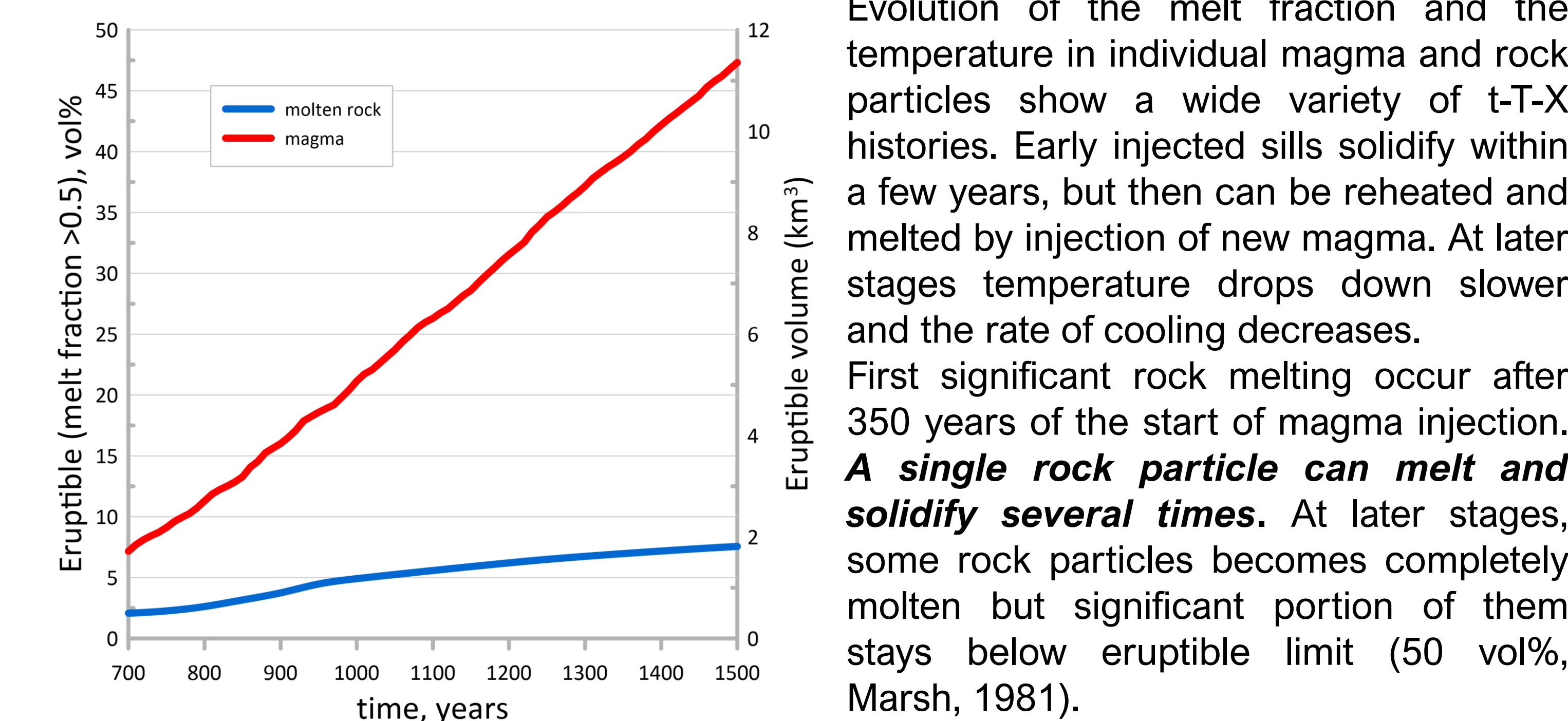
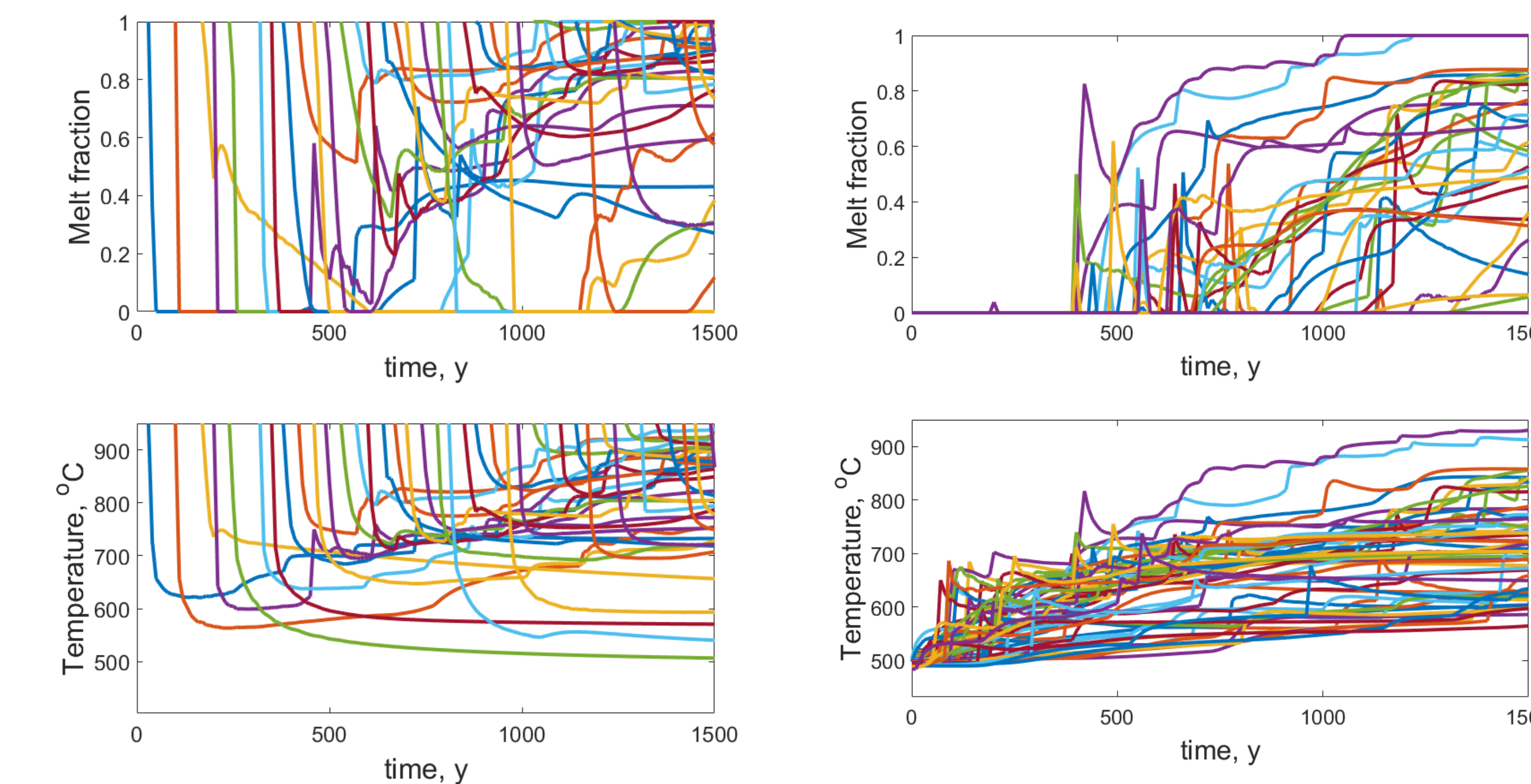
$L$  – latent heat of crystallization

Rock-magma temperature evolution is governed by heat conduction equation that accounts for advection due to rock displacement, latent heat of crystallization and heat conduction. Melt fraction depends on temperature according to Annen et al 2006. Injection rate is  $0.015 \text{ km}^3/\text{y}$  ( $0.5 \text{ m}^3/\text{s}$ ), initial magma temperature is  $950^\circ\text{C}$ , typical for an island arc system.

In order to minimize numerical diffusion PIC/FLIP hybrid method which mixes the perspectives of solving the system from a particle point of view (Lagrangian) and solving the system from a grid point of view (Eulerian) is used. Particle displacement is calculated after each individual dike injection, while heat conduction is solved on a fixed grid.

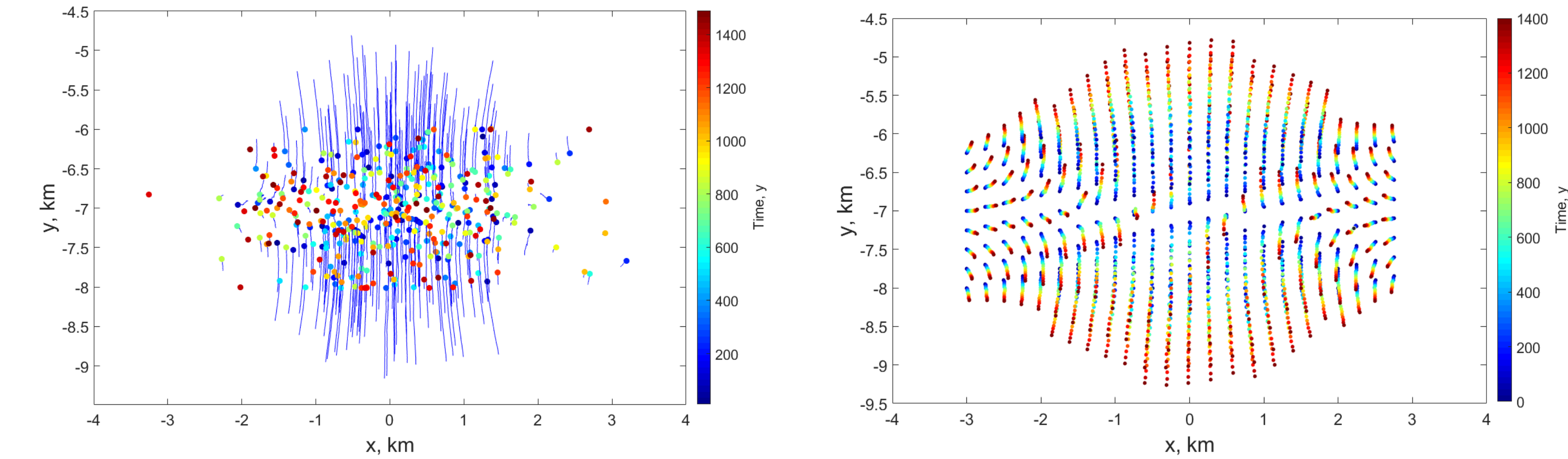


Initial injection of hot rhyolitic magma into silicic crust (same of slightly different T-x phase diagrams) incubate heat for 300 yrs without melting country rocks. Between 350 and 700 yrs more magma remains in molten state separated by both molten and solid country rocks. Individual weakly connected batches of melt are formed by 700 ys. At 1000 yrs most of magma batches become interconnected. At all times, the volume occupied by injected sills is significantly larger than the volume of molten rocks (Fig. below). Degree of partial melt vary laterally as many dikes stay solidified in the periphery of the magma chamber. If injection continues a body of completely molten rock is formed after 1500 y but current model cannot adequately simulate this stage of magma body evolution.



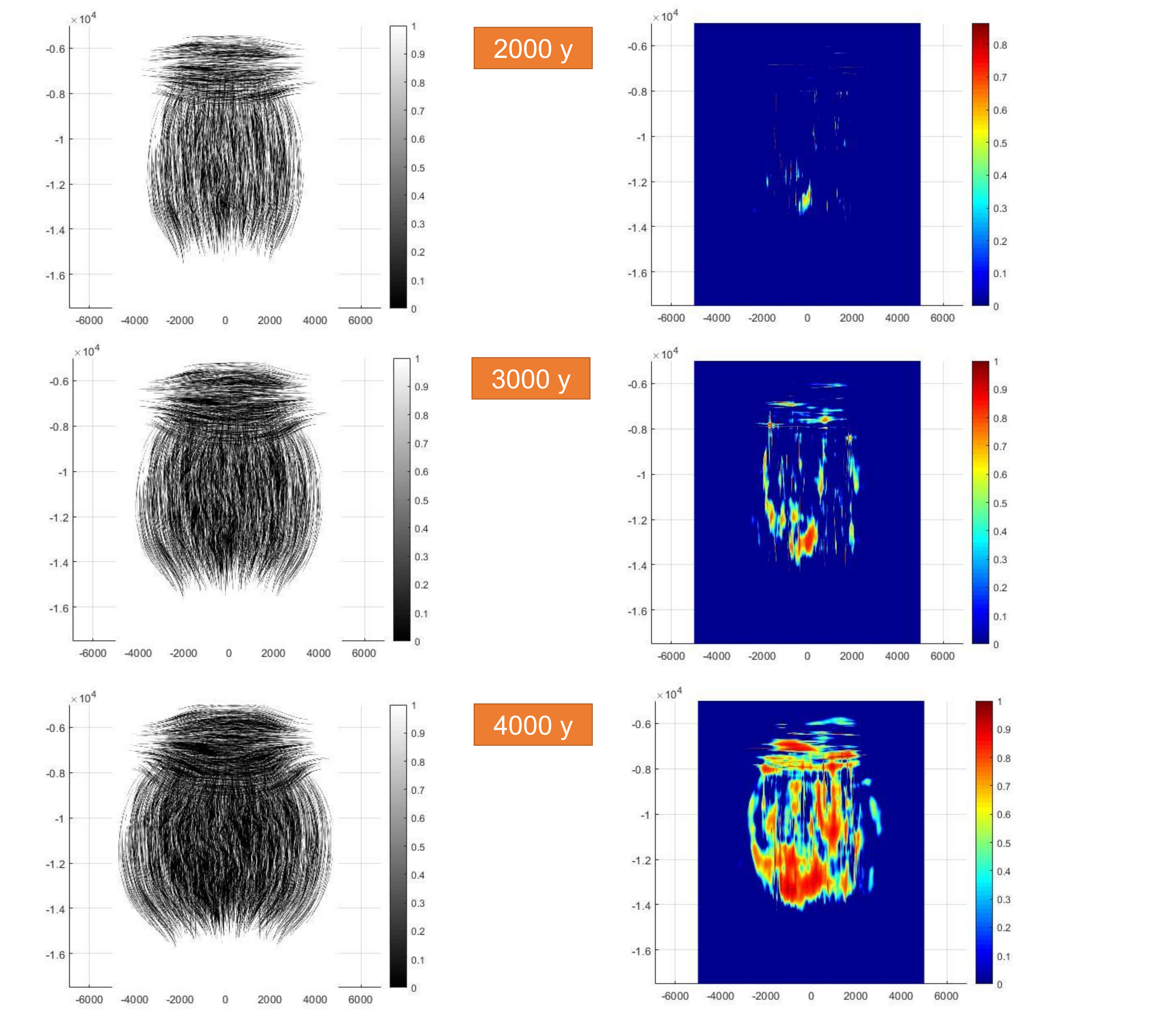
Evolution of the melt fraction and the temperature in individual magma and rock particles show a wide variety of t-T-X histories. Early injected sills solidify within a few years, but then can be reheated and melted by injection of new magma. At later stages temperature drops down slower and the rate of cooling decreases. First significant rock melting occur after 350 years of the start of magma injection.

**A single rock particle can melt and solidify several times.** At later stages, some rock particles becomes completely molten but significant portion of them stays below eruptible limit (50 vol%, Marsh, 1981).



Trajectories of individual magma parcels (left) and rock particles (right) in the case of sills injection. Trajectories are relatively simple – vertical extension dominates horizontal. **Rock particles in the Peripheral areas of the magma chamber move towards the center of the magma chamber because vertical expansion of the rock matrix leads to horizontal contraction.** Sills that intrude at late stages of magma chamber formation do not move far from their original location, while those intruded early are displaced by more than 2 km.

## Dike orientation effects



In general, the direction of dike propagation is vertical because it is perpendicular to the maximum principal stress. However, at the level of neutral buoyancy or when subsurface structure of rock contains layers with different mechanical properties, fractures propagate in horizontal direction (sills). The results of the simulated process of magma chamber formation where the magma intrusion occurs in the form of sills at depth in range of 6 to 4 km, and in the form of dikes at depth in range of 12 to 6 km are shown at the figure above.

Until 4000 years each magma portion solidifies quickly, and melt fraction remains low. Then, two areas enriched with melt begin to form: at a depth of 12-14 km where magma intrudes into the hottest rocks, and in shallow subsurface where the orientation of dikes changes. Between these areas the system of pathways develops through which magma can rise from deeper sources of melt.

## References

- Annen, C., Blundy, J. D., & Sparks, R. S. J. (2005). The genesis of intermediate and silicic magmas in deep crustal hot zones. *Journal of Petrology*, 47(3), 505-539.
- Marsh B.D. (1981) On the Crystallinity, Probability of Occurrence, and Rheology of Lava and Magma. *Contributions to Mineralogy and Petrology*. 78(1):85-98.
- Muskhelishvili, N. I. (2013). *Some basic problems of the mathematical theory of elasticity*. Springer Science & Business Media.