Discharge estimations of rivers from altimetry and datasets by hybrid computational methods

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Outline

▷The inverse problem to solve.

▷The hybrid (Artificial Neural Network - Variational Data Assimilation) algorithm.

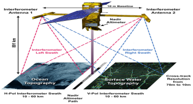
Estimation of the discharge $Q(x,t) \oplus$ the bathymetry $b(x)$ and roughness coeff. $K(x,h)$.

▷Numerical results : "Pepsi challenge" rivers.

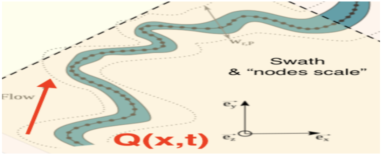
Application of the original HiVDI algo. (VDA w/o ANN) to anabranching river with lateral fluxes :
see talk presented by L. Pujol, P.-A. Garambois et al.

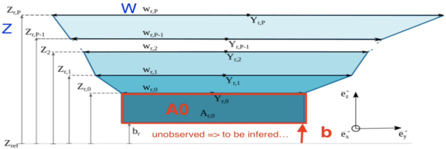
SWOT-like data & Inverse problem(s)

Forthcoming SWOT mission (NASA-CNES et al., to be launched in 2021)
Swath mapping radar interferometer (2x60 km). 1 to 4 revisits per 21 days repeat cycle



▷**Data :** water elevation Z & width W at nodes ($\sim$ 200m) scale
$\oplus$ slopes $\mathcal{S}$ at reach ($\sim 1 - 10$ km) scale





▷**Inverse problem** : Infer the discharge $Q_{r,p}$ at "reach" r, instant p.
$\Rightarrow$ To infer the bathymetry b_r ($\Leftrightarrow$ the lowest unobserved wetted cross section $A_{0,r}$)
$\oplus$ an effective spatially distributed roughness coefficient $K_r(h)$.
Water depth $h = (Z - b)$.

Hierarchical flow models

▷ $\boxed{\textbf{Dynamic model : Saint} - \textbf{Venant's equations}}$ 1D SWE set up at nodes scale ($\approx$ 200m).

$$\boxed{\partial_t(A,Q) + \partial_x(Q, Q^2/A)) + (0, gA\partial_x b) = (0, -gAS_f((K; A, Q)) \oplus Q_{in}(t) \text{ imposed at inflow.}}$$

Of course, outflow B.C. are part of the unknowns too (normal depth condition).

"Parameters" to be identified : $Q_{in}(t)$ inflow discharge $\oplus$ $b(x)$ effective bed elevation

$\oplus$ corresponding roughness coeff. $K(\cdot)$ or $\mathbf{K}(\alpha, \beta; h) = \alpha h^{\beta}$.

$\Rightarrow$ unknown "parameters" $c = (Q_{in}(t), b(x); (\alpha, \beta))$.

▷ $\boxed{\textbf{Dedicated algebraic (low complexity) 0.5D model.}}$

Local low Froude equilibriums (Manning-Strickler's laws) set up at reach scale ($\approx 1 - 10$km).

Considering P **overpasses**, the equations read :

$$(\mathcal{M}_2) : \quad \boxed{c_{r,p} \cdot K_{r,p}^{3/5} A_{r,0} + d_{r,p} \cdot K_{r,p}^{3/5} = Q_{r,p}^{3/5} \ \forall reach\ r\ \forall pass\ p}$$

with $(c_{r,p}, d_{r,p})$ given by the altimetry measurements

If considering the full set of unknowns $(K_{r,p}, A_{r,0}, Q_{r,p})$ then $(\mathcal{M}_2)$ is under-determined. But...

Use 1) Given $(Q_{r,p})$ e.g. by a Neural Network (see later)

$(\mathcal{M}_2)$ provides $(A_{r,0})_r$ with effective low-Froude friction coeff. $K_{r,p}$ $\forall r$ $\forall p$.

Use 2) Given $(A_{r,0}, K_{r,p})$ (estimations resulting of the inversions done during the "rivers learning period"),

$(\mathcal{M}_2)$ provides $Q_{r,p}$ $\forall r$ $\forall p$ in real-time. [Larnier-Monnier et al.] rev., to appear.

VDA formulation and ill-posedness

Recall. The unknown "control variable" in SWE is $c = (Q_{in}(t), b(x); (\alpha, \beta))$.

$\triangleright$**The inverse problem** reads : $\boxed{\min_c j(c)}$ with $j(c) = \|Z(c) - Z_{obs}\|_*^2$.

$\rightarrow$ Minimization by a gradient-based algorithm (L-BFGS) - adjoint method.

$\triangleright$**Probabilistic - physical a-priori introduced via covariance matrices**.

Change of variables : $k = B^{-1/2}(c - c_{prior})$ with $B = diag(B_Q, b_b, B_K)$.

The covariances matrices may be chosen as : $(B_\square)_{i,j} = (\sigma_\square)^2 \exp\left(-|x_j - x_i|/\mathbf{L}_\square\right)$

$\Rightarrow$ Parameters to be set : the 3 amplitudes $\sigma_\square$ and the 3 correlation scales $\mathbf{L}_\square$.

$\Rightarrow$ **Priors of the VDA formulation :**

- The first guess values : $c^{(0)} = \left(Q_{in}^{(0)}(t), b^{(0)}(x); (\alpha, \beta)^{(0)}\right)$.
- The regularization parameters $\sigma_\square$ and $\mathbf{L}_\square$.

$\triangleright$Based on the flow models only, the inverse problems is ill-posed :
the unknowns $c = (Q_{in}(t), b(x); (\alpha, \beta))$ **may be unique up to a scalar scaling factor only.** [Larnier-Monnier] submitted.

$\Rightarrow$ **Priors of the VDA formulation select the (physically-consistent) solution therefore the estimations of** $Q_{r,p}$ **...**

Hybrid ANN - VDA inversions

▷**Stage 1) From datasets (e.g. Pepsi1, 2 and HydroSWOT), train an Artificial Neural Network (ANN) to estimate** $Q_{r,p}$.

Case In) Rivers inside the learning flow regimes : excellent estimations.

Case Out) Rivers outside the learning flow regimes, bad estimations but (crucial) space-time variations are captured.

$\Rightarrow$ **First guess value** $Q^{(0)} \equiv Q^{(ANN)}$.

▷**Stage 2)**

a) Algebraic model with $Q^{(ANN)}$ given $\Rightarrow$ First guess values $\left(b_r^{(0)}; K_r(h)^{(0)}\right) \forall r$.

b) VDA based inversions

Estimation of the complete set (discharge $Q_{r,p}$, bathymetry b_r and roughness $K_r(h)$).

$\Rightarrow$ **The estimated** $Q_{r,p}$ **is accurate, at worse with a low bias**.

Estimations obtained at fine scale i.e. at node scale $\sim$ 200 m. Ref. [Larnier-Monnier] sub.

▷**Stage 3)** Past this river learning period (e.g. one year), given newly acquired SWOT measurements $(Z, W)_{r,p}$,

$\Rightarrow$ **Estimations of** $Q_{r,p}$ **in real-time** using the (low complexity) algebraic system $(\mathcal{M}_2)$. Inference obtained at reach scale ($\sim 1 - 10$ km). Ref. [Larnier-Monnier et al.] Rev., to appear.

NB. Possible to assimilate any existing global-regional database values (if reliable).

Numerical results

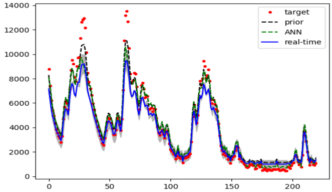
▷**SWOT-like data only : no additional in-situ information ; fully ungauged rivers**.
Pepsi 1 and 2 datasets : 27 rivers portions presenting very wide ranges of flow regimes.
Q within $\approx [10^2, 10^5]\ m^3/s$.

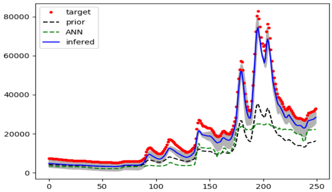
- Cal-Val SWOT frequency : 1 day revisit.
- SWOT-like data = models outputs (from HEC-Ras, DassFlow) $\oplus$ realistic Gaussian noise $\Rightarrow$ **Target values**.

▷**Rivers partly within the ANN-learned flow regimes** e.g. Fig. (Left) : Ohio river.

- The ANN estimation is very good.
- Next, the VDA estimation may be better, similar or slightly less accurate.

Present case : the VDA estimation is a bit less accurate than the ANN one.





▷**Rivers far outside the ANN-learned flow regimes** e.g. Fig. (Right) : Jamuna river.

- The algebraic flow model $\mathcal{M}_2$ (="prior" in Fig.) improves the accuracy of $Q^{(ANN)}$.
- The VDA estimation (="infered" in Fig.) provides a relatively good estimation of $Q(x,t)$.

All other rivers (27 in total) have been analysed : similar results.

Conclusion

▷The (purely physically informed) VDA approach provides accurate space-time variations of $Q(x,t)$ (at the observations "hours scale") *but up to an intrinsic bias*.

(At least if based on the usual flow models...).

▷**Combining the VDA inversions with ANN** makes greatly decrease the bias.

$\Rightarrow$ **Much more reliable estimations of $Q(x,t)$.**

Accurate space-time variations ; if a bias remains it is observed to be relatively low.

▷Past the "learning period", **real-time estimations of $Q_{r,p}$** are possible using the dedicated low-complexity algebraic flow model.

▷This algorithm (named H2iVDI) can be naturally applied to **multi-satellites data**.

See e.g. PhD T. Malou (IMT-INSA - INRAE - CLS group) in progress.

▷**The HiVDI algorithm has been applied** by the colleagues from Univ. Strasbourg and INRAE

to anabranching rivers and with coupling to hydrological models. See talk presented by L. Pujol et al.

References

- K. Larnier, J. Monnier. "Hybrid estimations of river discharges from altimetry". Sub.
- K. Larnier, J. Monnier, P.-A. Garambois, J. Verley. "On the estimation of river discharges from altimetry". In rev. (to appear).
- P. Brisset, J. Monnier, P.-A. Garambois, H. Roux. "On the assimilation of altimetry data in 1D Saint-Venant river models". Adv. Water Ress., 2018.

HiVDI algorithms are available in DassFlow, open source computational software. Test it !

http ://www.math.univ-toulouse.fr/DassFlow