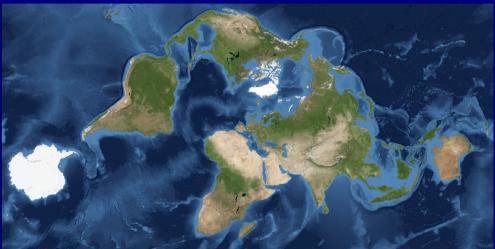
Optimized global map projections for specific applications: The triptychial projection and the Spilhaus projection Björn Grieger, European Space Astronomy Centre, Madrid



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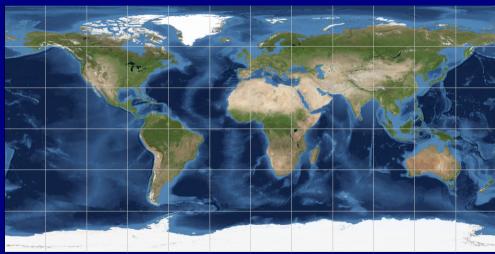
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Equidistant cylindrical projection





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Neither shape nor area preserving.

- ► Widely used because of apparent simplicity.

"Straight line" (great circle) in the real world



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Beware: straight lines are not straight lines!
 ...e.g., your flight from Madrid to Los Angeles.
 However, a great format to exchange surface data.

Mercator projection



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 Conformal, i. e., shape preserving on

 Large scale distorsions (Greenland appears larger than Africa), poles are at

Previously used by Google

Never intended for global

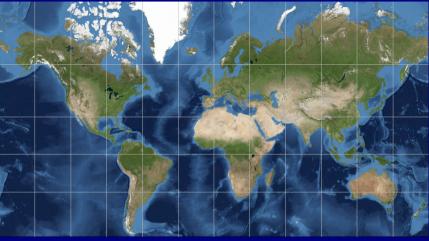
infinity.

maps.

world maps.

(infinitesimal) small scales.

Abused as world map



- ► Very bad polar areas just cut off.
- Saw it with Greenland photoshopped out at the wall of a travel agency office (they obviously didn't sell trips to Greenland).



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Really the seafarer's map





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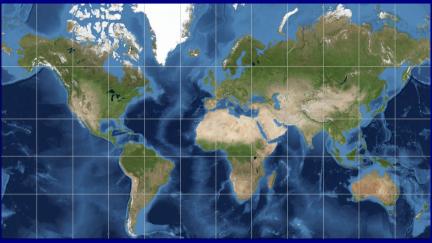
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► The **real** purpose is navigation.

Steering a fixed course comes out as straight line!

If size matters ...



 Criticized for showing countries near the Equator as too small when compared to Europe and North America.



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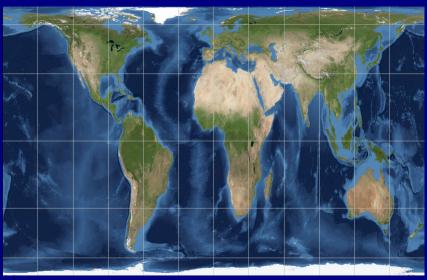
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Gall-Peters projection



▶ Perfectly area preserving, but large shape distortions.



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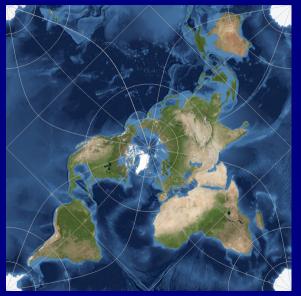
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Peirce quincuncial projection (1879)





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Equidistant cylindrical Mercator projection

Peirce guincuncial projection

treated nicely! (Peirce said that nobody lives there, so

Conformal (with the

Approximately area

preserving over the

... but Antarctica is not

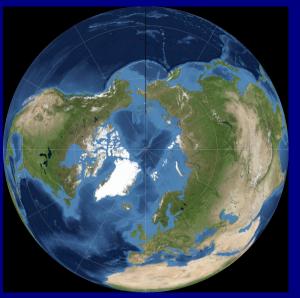
nobody will complain.)

the edges).

continents.

exception of four singular points at the centers of

The Northern hemisphere





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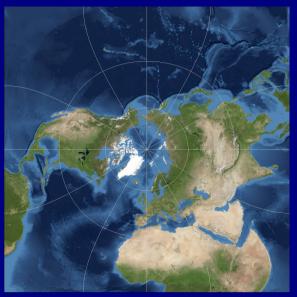
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... conformally mapped to a square





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Elliptic isometric coordinates

Longitude
$$\lambda$$
, latitude φ , elliptic integral $F(\alpha) = \int_0^{\alpha} \frac{1}{\sqrt{1 - \frac{1}{2}\sin^2 \alpha'}} d\alpha'$:

$$\begin{aligned} \xi_1 &= \arccos\left(\cos\varphi\,\cos\left(\frac{\pi}{4} + \lambda\right)\right) & \eta_1 &= \arccos\left(\cos\varphi\,\sin\left(\frac{\pi}{4} + \lambda\right)\right) \\ \xi_2 &= \arcsin\left(\sqrt{2}\,\cos\left(\frac{\xi_1 + \eta_1}{2}\right)\right) & \eta_2 &= \arcsin\left(\sqrt{2}\,\sin\left(\frac{\xi_1 - \eta_1}{2}\right)\right) \\ x &= \operatorname{sgn}(\xi_2)\,F(|\xi_2|) & y &= \operatorname{sgn}(\eta_2)\,F(|\eta_2|) \\ x &\in [-1.84533, 1.84533] & y &\in [-1.84533, 1.84533] \end{aligned}$$

This maps a point (λ, φ) on the Northern hemisphere to a point in a square with rectangular coordinates (x, y).



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Two hemispheres mapped one by one

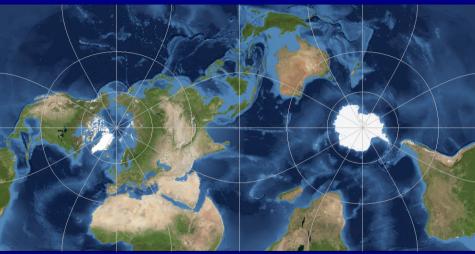


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Equidistant cylindrical

Conformal mapping of a hemisphere to a square



- ▶ The two hemispheres shown side-by-side.
- Conformal with the exception of the four corner points.
- Area distortions get large close to these points.

Slightly traverse version



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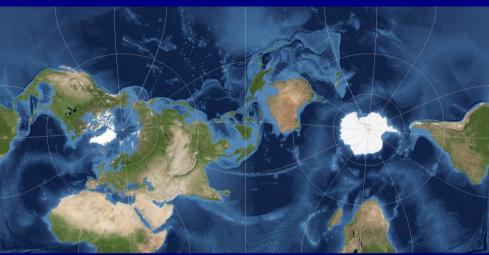
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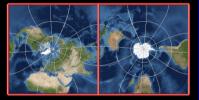
References



Rotated 25° around the z-axis to get the continents away from the critical points.

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Two hemispheres side by side.





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Two hemispheres side by side.

This can be tessellated.





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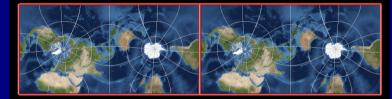
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Two hemispheres side by side.

This can be tessellated.



The short edges match perfectly.



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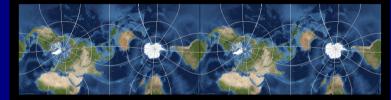
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Two hemispheres side by side.

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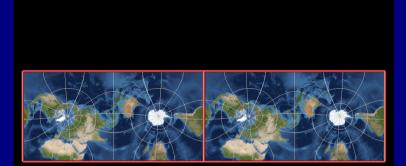
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The short edges match perfectly.



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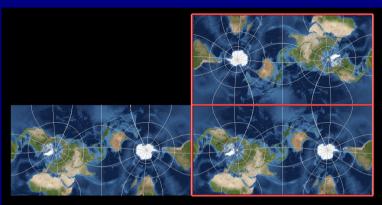
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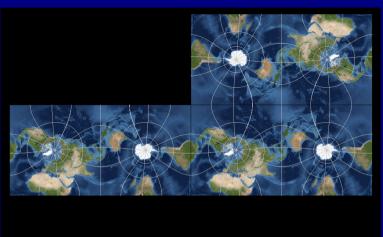
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The short edges match perfectly.

The long edges match after a rotation by 180°.





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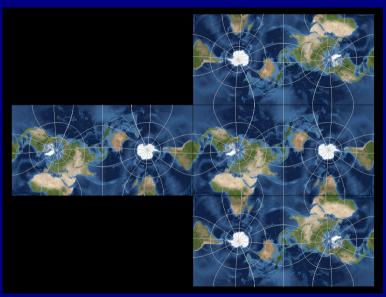
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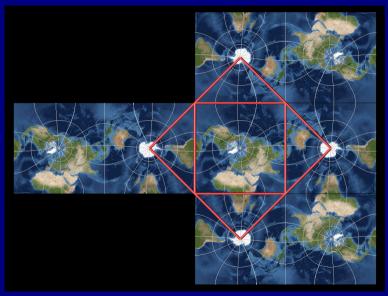
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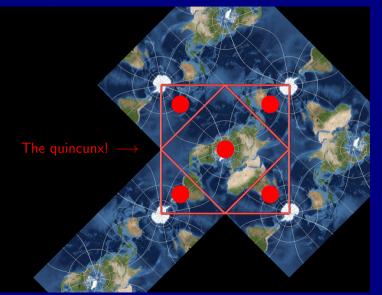
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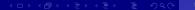
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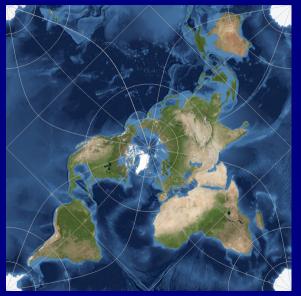
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Peirce quincuncial projection (1879)





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Conformal (with the

 Approximately area preserving over the

 ... but Antarctica is not treated nicely! (Peirce said that nobody lives there, so nobody will complain.)

the edges).

continents.

exception of four singular points at the centers of

The triptychial projection (Grieger, 2019)



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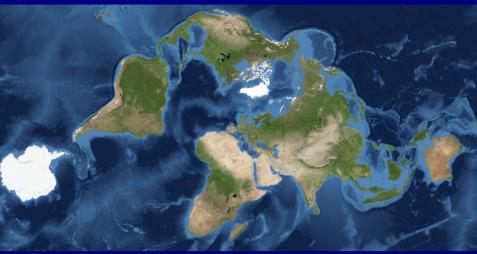
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A map of the whole world showing all continents including Antarctica with minimal distortion and without any intersection

Standard "quincuncial" projection



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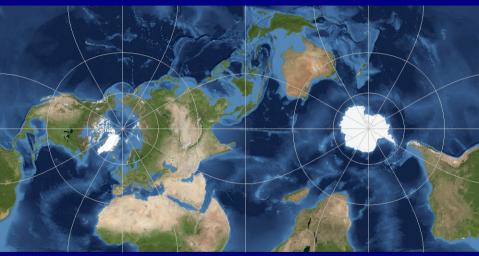
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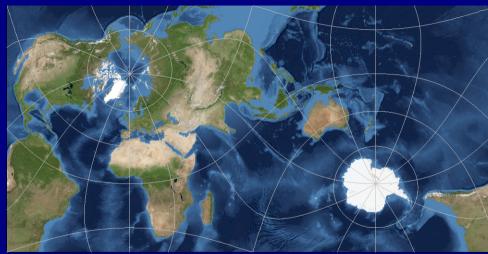
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Rotated 45° counterclockwise around the *y*-axis ...





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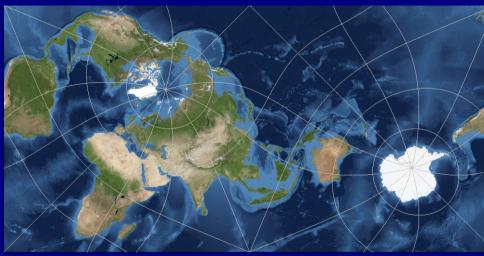
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... and 45° counterclockwise around the z-axis





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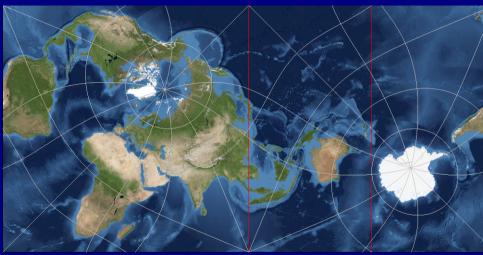
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Cut the right hemisphere into two halfs





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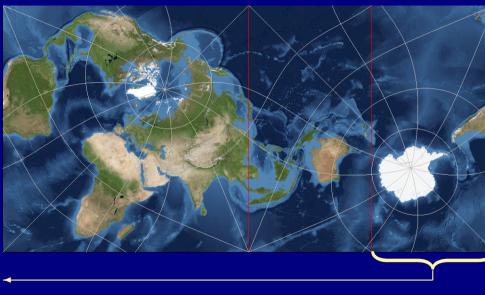
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... and move the right half over to the very left





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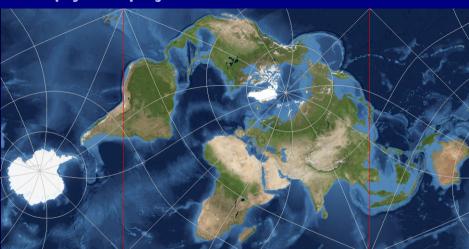
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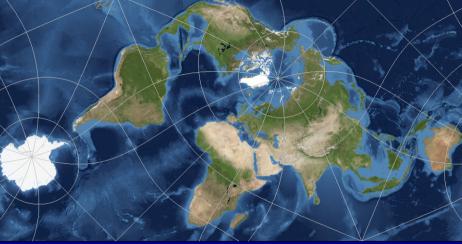
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- Conformal (shape preserving on small scales).
- Approximately area preserving over the continents.

The triptychial projection (Grieger, 2019)



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Triptychial layout

Equidistant cylindrical

A map of the whole world showing all continents including Antarctica with minimal distortion and without any intersection

Surface data



So we have a mapping:

 $\lambda, \varphi \longrightarrow \mathsf{Surface data}$

Note: Center of pixel (1, 1):

$$\lambda = -180 \, \frac{n - \frac{1}{2}}{n}, \varphi = 90 \, \frac{\frac{n}{2} - \frac{1}{2}}{\frac{n}{2}}$$

Center pixel (2n, n):

$$\lambda = 180 \, \frac{n - \frac{1}{2}}{n}, \varphi = -90 \, \frac{\frac{n}{2} - \frac{1}{2}}{\frac{n}{2}}$$

(1)

Rectangular map with $2n \times n$ pixels:

 $i_x, i_y \longrightarrow$ Surface data

Equidistant cylindrical projection:

 $\begin{array}{c} \lambda \longleftrightarrow i_{\mathsf{X}} \\ \varphi \longleftrightarrow i_{\mathsf{V}} \end{array}$

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Applying the projection

Triptychial projection:

$$\lambda, \varphi \longrightarrow j_x, j_y$$

Direct application is not recommened, as it makes trouble at the edges of the map. Instead, use the inverse (needs numerical inversion):

$$j_x, j_y \longrightarrow \lambda, \varphi$$

For each pixel (j_x, j_y) of the projected map:

- **1.** Get (λ, φ) from the mapping (2).
- 2. Get the surface data from the mapping (1).

Readymade tables providing mapping (2) are available online from the author, see <u>References</u>.



(2)

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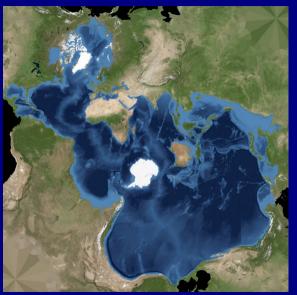
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The Spilhaus projection (1979, not 1942)



A map showing the whole world ocean without any intersection and with only moderate distortion



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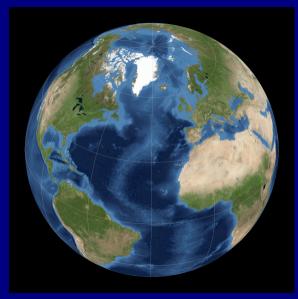
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The whole sphere ...



Longitude λ_1 , latitude φ_1

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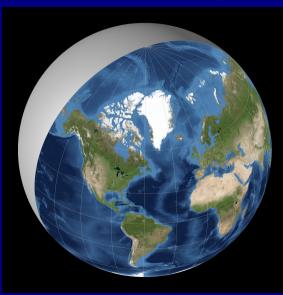
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... contracted conformally to a hemisphere ...



New longitude and latitude:

$$\lambda_2 = \frac{1}{2} \lambda_1$$

$$\varphi_2 = \frac{\pi}{2} - \frac{1}{2} - \frac{1}{2} + \frac{1}{2}$$

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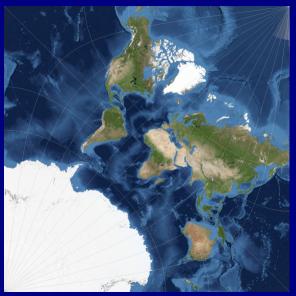
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... and then conformally mapped to a square



Like before one hemisphere for the quincuncial projection:

 $(\lambda_2, \varphi_2) \longrightarrow (x, y)$

But here the hemispere contained already the whole (contracted) world!



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The Adams projection of the world in a square II $(1929)^{\bigcirc}$

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1. Rotate the poles well into China (near Hankou) and South America (near Cordoba):

 -60° around (cos 205°, sin 205°, 0)

2. Rotate the point $(169^{\circ}W, 65.3^{\circ})$ near the Bering Strait to the edge:

 -88.02° around the new *z*-axis



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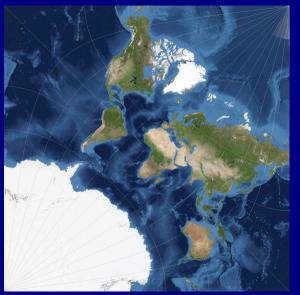
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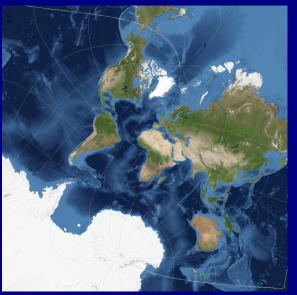
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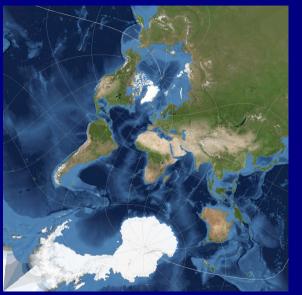
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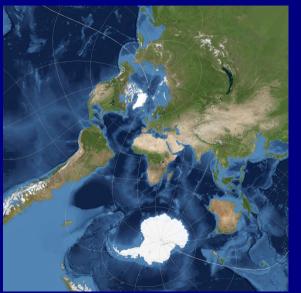
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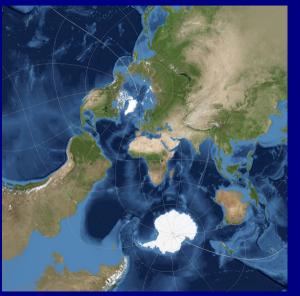
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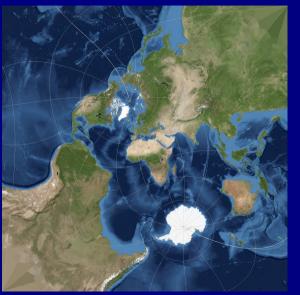
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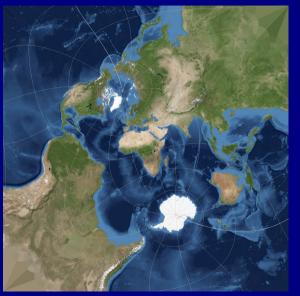
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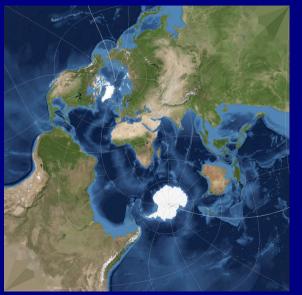
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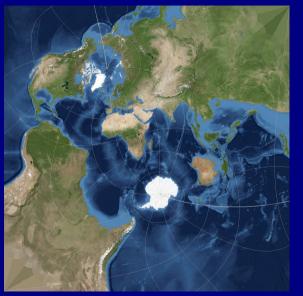
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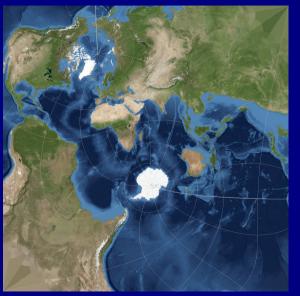
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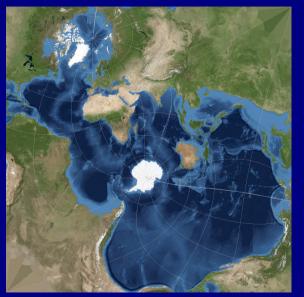
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Widened view (replications from tesselation)





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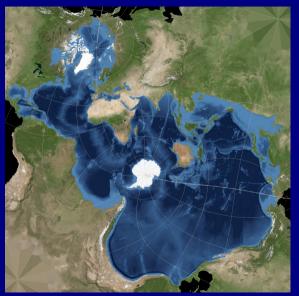
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Replicated ocean blacked out





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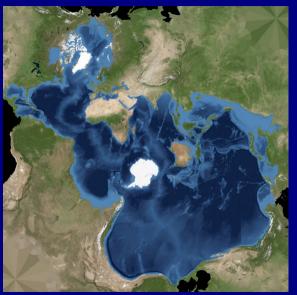
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- ► Tables to do it yourself:

http://comsim.esac.esa.int/rossim/bgrieger/triptychial



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Surface data used

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in NASA's Blue Marble collection at

https://visibleearth.nasa.gov/collection/1484/blue-marble



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