

INTERACTION OF TEMPERATURE- AND SALINITY-DRIVEN NATURAL CONVECTION IN HOMOGENEOUS POROUS MEDIA



Endowed Hydrogeology Chair and Foundation





Márk Szijártó^{1,2}

(mark.szijarto@ttk.elte.hu)

Attila Galsa¹

- (1) Eötvös Loránd University, Department of Geophysics and Space Science
- (2) József and Erzsébet Tóth Endowed Hydrogeology Chair, Budapest, Hungary





INTRODUCTION



Onset of thermal and haline convection was studied **separately** by Lapwood (1948) and Wooding (1956) in theoretical models using analytical methods. They established that the buoyancy force caused by temperature (ΔT) and concentration difference (Δc) can induce natural convection. In this study, the combined effect of temperature- and salinity-driven natural convection was examined in 2D homogeneous porous media in the cases of three scenarios (Model A, B, C).

THE MAIN QUESTIONS

- How does the interaction of the thermal and haline term affect the onset of the natural convection?
- Under what conditions does a time-dependent flow system evolve in the theoretical models?
- How can the thermohaline convection be characterized by the nondimensional numbers?



Physical background

Partial differential equation system

- Mass conservation
- Darcy's Law
- Mass transport
- Heat transport

The equations were coupled by the Darcy flux (q) and the temperature- and concentrationdependent water density $(\rho_w(c,T))$.



Definition	Symbol	Value	Unit
Non-dimensional thermal expansion	$\alpha\Delta T$	0.01-1	-
Non-dimensional haline concentration	$\beta\Delta c$	10 ⁻⁵ -10 ⁻³	-
Permeability	k	10-11	m ²
Reference water density	ρ ₀	1000	kg/m ³
Model thickness	Н	10	m
Dynamic viscosity of water	η	0.001	Pa·s
Thermal diffusivity	κ	7.16·10 ⁻⁷	m²/s
Diffusion coefficient	D_0	10-9	m²/s







Non-dimensional thermal expansion and haline concentration were increased from $\alpha\Delta T$ =0.01 to 1 and from $\beta\Delta c$ =10⁻⁵ to 10⁻³ respectively, while the average Darcy flux (q_{av}) the Nusselt number (Nu) and the Sherwood number (Sh) were computed. Other physical properties (e.g. thickness (*H*=10 m), length (*L*=200 m), permeability (*k*=10⁻¹¹ m²) of the model domain were kept constant values.

Variable density of the pore water:

$$\rho_w(c,T) = \rho_0[1 + \beta c - \alpha T]$$



NON-DIMENSIONAL NUMBERS

Thermal Rayleigh number

$$\operatorname{Ra}_{T} = \frac{\mathrm{g}k\rho_{0}H}{\eta}\frac{\alpha\Delta T}{\kappa}$$

Lewis number

$$Le = \frac{\kappa}{D_0}$$

Buoyancy ratio

$$BR = \frac{\beta \Delta c}{\alpha \Delta T}$$

Haline Rayleigh number

$$\operatorname{Ra}_{H} = \operatorname{BR} \cdot \operatorname{Le} \cdot \operatorname{Ra}_{T} = \frac{gk\rho_{0}H}{\eta}\frac{\beta\Delta c}{D_{0}}$$

Thermohaline Rayleigh number

$$\operatorname{Ra}_{TH} = \frac{\mathrm{g}k\rho_0 H}{\eta} \left[\frac{\alpha \Delta T}{\kappa} + \frac{\beta \Delta c}{D_0} \right]$$

Region of the stability

$$\operatorname{Ra}_T + \operatorname{Ra}_H = 4\pi^2$$

(Nield and Bejan, 2006)

One of the most important questions is whether the previous formulas are universal. If this is not the case, how could the flow be easily characterized by the non-dimensional numbers?



THREE MODEL SCENARIOS







RESULTS – MODEL A Concentration Temperature **Initial conditions** $Ra_{TH} = 23.51$ $\log_{10}\alpha = -2$ $\log_{10}\beta$ =-5 $\log_{10}\alpha = -2$ $Ra_{TH} = 111.9$ $\log_{10}\beta = -4$ $\log_{10}\alpha = -1$ Ra_{TH}=148.8 $\log_{10}\beta = -5$ $Ra_{TH} = 238.1$ $\log_{10}\alpha = -1$ $\log_{10}\beta=-4$ Ra_{TH}=994.7 $\log_{10}\beta$ =-3 $\log_{10}\alpha = -2$ 6/12 *t*=10000 d 0 0 1





- \succ αΔ*T*<=10⁻² and βΔ*c*<=10⁻⁵ → **no thermal and haline convection** in the porous medium
- \blacktriangleright αΔ*T*<=10⁻² and βΔ*c*=10⁻⁴−10⁻³ → steady-state and time-dependent **haline convection** without any thermal effects
- \succ αΔ*T*=10⁻¹ and βΔ*c*=10⁻⁵−10⁻³ → steady-state **thermohaline convection**
- \succ αΔ*T*=1 and βΔ*c*=10⁻⁵−10⁻³ → time-dependent **thermohaline convection**

The effect of salinity-driven convection was strongly influenced by the heat transport mechanisms, because the Lewis number was Le=716. However, the natural convection might not be universally characterized by the thermohaline Rayleigh number!?



RESULTS – MODEL A







994.7

 10^{-1}



Results – Model B



No convection Steady-state thermohaline convection Time-dependent thermohaline convection Ra_{TH} - Thermohaline Rayleigh number







Results – Model C





SUMMARY & OPEN-ENDED QUESTIONS

Model A

m

Model

C

Model

Both effects facilitated the onset of natural convection. However, the effect of salinity-driven natural convection was strongly influenced by the heat transport mechanisms ($k > D_0$). Steady-state haline, thermohaline and time-dependent thermohaline convection were noticed (e.g. Nu vs BR, Sh vs BR).

The natural convection was facilitated by thermal effect, but it was reduced by saline effect. In this case, steady-state and time-dependent thermohaline convection were noticed in the numerical model. The onset of thermohaline convection might be defined by a critical Rayleigh number (Ra_{TH}) with a buoyancy ratio (BR).

The natural convection was facilitated by saline effect, but it was reduced by thermal effect. In this case, only haline convection evolved in the numerical model, which was shown by the Darcy flux $(q_{av}>10^{-10})$, the Sherwood (Sh>1) and the Nusselt number (Nu=1).

- How to define the thermohaline Rayleigh number in the different model scenarios?
- > What is the critical value of the thermohaline Rayleigh number?
- Maybe a new non-dimensional number should be defined in order to characterize the thermohaline convection?





ACKNOWLEDGEMENTS





This project was supported by the:

- ÚNKP-19-3 and ÚNKP-19-4 New National Excellence Program of the Ministry for Innovation and Technology
- Hungarian Scientific Research Fund (OTKA/K 129279)
- > János Bolyai Research Scholarship of the Hungarian Academy of Science
- József and Erzsébet Tóth Endowed Hydrogeology Chair

This research is part of a project that has received funding from the **European Union's Horizon 2020 research and innovation program** under grant agreement No 8130980.





Endowed Hydrogeology Chair and Foundation







12/12