Multi-fidelity approach to Bayesian parameter estimation in subsurface heat and fluid transport models

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The increased use of the urban subsurface for multiple purposes, such as anthropogenic infrastructures and geothermal energy applications, leads to an urgent need for large-scale sophisticated modelling approaches for coupled mass and heat transfer. However, such models are subject to large uncertainties in model parameters, the physical model itself and in available measured data, which is often rare. Thus, the robustness and reliability of the computer model and its outcomes largely depend on successful parameter estimation and model calibration, which are often hampered by the computational burden of large-scale coupled models.

To tackle this problem, we present a novel Bayesian approach for parameter estimation, which allows to account for different sources of uncertainty, is capable of dealing with sparse field data and makes optimal use of the output data from computationally expensive numerical model runs. This is achieved by combining output data from different models that represent the same physical model, but at different levels of fidelity, e.g. reflected by different spatial resolution, i.e. different model discretization. Our framework combines information from a few parametric model outputs from a physically accurate, but expensive, high-fidelity computer model, with a larger number of evaluations from a less expensive and less accurate low-fidelity model. This enables us to include accurate information about the model output at sparse points in the parameter space, as well as dense samples across the entire parameter space, albeit with a lower physical accuracy.

We first apply the multi-fidelity approach to a simple 1D analytical heat transfer model, and secondly on a semi-3D coupled mass and heat transport numerical model, and estimate the unknown model parameters. By using synthetic data generated with known parameter values, we are able to test the reliability of the new method, as well as the improved performance over a single-fidelity approach, under different framework settings. Overall, the results from the analytical and numerical model show that combining 50 runs of the low resolution model with data from only 10 runs of a higher resolution model significantly improves the posterior distribution results, both in terms of agreement with the true parameter values and the confidence interval around this value. The next steps for further testing of the method are
employing real data from field measurements and adding statistical formulations for model calibration and prediction based on the inferred posterior distributions of the estimated parameters.