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Nonlinear Fourier Analysis for Two-Dimensional Ocean Surface Waves Described by the Zakharov Equation

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The physical hierarchy of two-dimensional ocean waves studied here consists of the 2+1 nonlinear Schrödinger equation (NLS), the Dysthe equation, the Trulsen-Dysthe equation, etc. on to the Zakharov equation. I call this the SDTDZ hierarchy. I demonstrate that the nonlinear Schrödinger equation with arbitrary potential is the natural way to treat this hierarchy, for any member of the hierarchy can be determined by an appropriate choice of the potential. Furthermore, the NLS equation with arbitrary potential can be written in terms of two bilinear forms and thereby has one and two-soliton solutions. To access the inverse scattering approach, I find a nearby equation which has N-soliton solutions: Such an equation is completely integrable by the IST on the infinite plane and by finite gap theory for periodic boundary conditions. In this way the entire SDTDZ hierarchy is closely related to a nearby integrable hierarchy which I refer to as the iSDTDZ hierarchy. Every member of this hierarchy has solutions in terms of ratios of Riemann theta functions and therefore every member has general spectral solutions in terms of quasiperiodic Fourier series. This last step occurs because ratios of theta functions are single valued, multiply periodic meromorphic functions. Once the quasiperiodic Fourier series are found, one can then invert these to determine the Riemann spectrum, namely, the Riemann matrix, wavenumbers, frequencies and phases. This means that the solutions of the nonlinear wave equations of the iSDTDZ hierarchy are generalized Fourier series indistinguishable from those of Paley and Weiner [1935] and therefore allows one to classify nonlinear wave motion in terms of a linear superposition of sine waves. How do the generalized quasiperiodic Fourier series differ from ordinary, standard periodic Fourier series? This can be seen by recognizing that the frequencies are incommensurable, and the phases can be phase locked. The nonlinear Fourier modes are Stokes waves and the coherent structure solutions are nonlinearly interacting, phase-locked Stokes waves, including breathers and superbreathers. Other types of coherent packets include fossil breathers and dromions. Techniques are developed for (1) numerical modeling of ocean waves (a fast algorithm for the Zakharov equation) and for (2) the nonlinear Fourier analysis of two-dimensional measured wave fields and space/time series (a 2D nonlinear Fourier analysis, implemented as a fast algorithm called the 2D NFFT). Examples of both applications are discussed.