Two sides of the same inverse problem, in identifying the pore size distribution based on experiments with non-Newtonian fluids

Martin Lanzendörfer
Charles University, Faculty of Science, Institute of Hydrogeology, Engineering Geology and Applied Geophysics, Praha 2, Czechia (martin.lanzendorfer@natur.cuni.cz)

Following the capillary bundle concept, i.e. idealizing the flow in a saturated porous media in a given direction as the Hagen-Poiseuille flow through a number of tubular capillaries, one can very easily solve what we would call the forward problem: Given the number and geometry of the capillaries (in particular, given the pore size distribution), the rheology of the fluid and the hydraulic gradient, to determine the resulting flux. With a Newtonian fluid, the flux would follow the linear Darcy law and the porous media would then be represented by one constant only (the permeability), while materials with very different pore size distributions can have identical permeability. With a non-Newtonian fluid, however, the flux resulting from the forward problem (while still easy to solve) depends in a more complicated nonlinear way upon the pore sizes. This has allowed researchers to try to solve the much more complicated inverse problem: Given the fluxes corresponding to a set of non-Newtonian rheologies and/or hydraulic gradients, to identify the geometry of the capillaries (say, the effective pore size distribution).

The potential applications are many. However, the inverse problem is, as they usually are, much more complicated. We will try to comment on some of the challenges that hinder our way forward. Some sets of experimental data may not reveal any information about the pore sizes. Some data may lead to numerically ill-posed problems. Different effective pore size distributions correspond to the same data set. Some resulting pore sizes may be misleading. We do not know how the measurement error affects the inverse problem results. How to plan an optimal set of experiments? Not speaking about the important question, how are the observed effective pore sizes related to other notions of pore size distribution.

All of the above issues can be addressed (at least initially) with artificial data, obtained e.g. by solving the forward problem numerically or by computing the flow through other idealized pore geometries. Apart from illustrating the above issues, we focus on two distinct aspects of the inverse problem, that should be regarded separately. First: given the forward problem with $N$ distinct pore sizes, how do different algorithms and/or different sets of experiments perform in identifying them? Second: given the forward problem with a smooth continuous pore size distribution (or, with the number of pore sizes greater than $N$), how should an optimal representation by $N$ effective pore sizes be defined, regardless of the method necessary to find them?