EGU21-7543, updated on 10 Dec 2022
https://doi.org/10.5194/egusphere-egu21-7543
EGU General Assembly 2021

Higher order phase averaging for big timesteps

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We introduce a higher order phase averaging method for nonlinear PDEs. Our method is suitable for highly oscillatory systems of nonlinear PDEs that generate slow motion through resonance between fast frequencies, such as is the case for rotating fluids with small but finite Rossby number. Phase averaging is a technique to filter fast motions from the dynamics whilst still accounting for their effect on the slow dynamics. In the small Rossby number limit of the phase averaged rotating shallow water equations, one recovers the quasi-geostrophic equations (as shown by Schochet, Majda and others). Peddle et al. 2017, Haut and Wingate 2014, have shown that phase averaging at finite Rossby number allows to take larger timesteps than would otherwise be possible. This was used as a coarse propagator (large timesteps at lower accuracy) for a Parareal method where corrections were made using a standard timestepping method with small timesteps.

In this contribution, we introduce an additional phase variable in the exponential time integrator that allows us to derive arbitrary order averaging methods that can be used as more accurate corrections to the basic phase averaged model, without needing small timesteps. We envisage their use as part of a time-parallel algorithm based on deferred corrections to the basic average. We illustrate the properties of this method on an ODE that describes the dynamics of a swinging spring, a model due to Peter Lynch. Although idealized, this model shows an interesting analogy to geophysical flows as it exhibits a high sensitivity of small scale oscillation on the large scale dynamics. On this example, we show convergence to the non-averaged (exact) solution with increasing approximation order also for finite averaging windows. At zeroth order, our method coincides with that in Peddle et al. 2017, Haut and Wingate 2014, but at higher order it is more accurate in the sense that it better approximates the faster oscillations around the slow manifold.