Budyko-Sellers 2.0: the classical and fractional heat equations, and the fractional energy balance equation

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The highly successful Budyko-Sellers energy balance models are based on the classical continuum mechanics heat equation in two spatial dimensions. When extended to the third dimension using the correct conductive-radiative surface boundary conditions, we show that surface temperature anomalies obey the (nonclassical) Half-order energy balance equation (HEBE, with exponent $H = \frac{1}{2}$) implying heat is stored in the subsurface with long memory.

Empirically, we find that both internal variability and the forced response to external variability are compatible with $H \approx 0.4$. Although already close to the HEBE and classical continuum mechanics, we argue that an even more realistic “effective media” macroweather model is a generalization: the fractional heat equation (FHE) for long-time (e.g. monthly scale anomalies). This model retains standard diffusive and advective heat transport but generalize the (temporal) storage term. A consequence of the FHE is that the surface temperature obeys the Fractional EBE (FEBE), generalizing the HEBE to $0 < H \leq 1$. We show how the resulting FEBE can be been used for monthly and seasonal forecasts as well as for multidecadal climate projections. We argue that it can also be used for understanding and modelling past climates.