Weak turbulence and quasilinear diffusion for relativistic wave-particle interactions via a Markov approach

Oliver Allanson\textsuperscript{1}, Thomas Elsden\textsuperscript{2}, Clare Watt\textsuperscript{3}, and Thomas Neukirch\textsuperscript{4}

\textsuperscript{1}Environmental Mathematics Group & Geophysical and Astrophysical Fluid Dynamics Group, Department of Mathematics, University of Exeter, Penryn/Cornwall Campus, United Kingdom
\textsuperscript{2}School of Mathematics and Statistics, University of Glasgow, Glasgow, United Kingdom
\textsuperscript{3}Department of Mathematics, Physics and Electrical Engineering, Northumbria University, Newcastle upon Tyne, United Kingdom
\textsuperscript{4}School of Mathematics and Statistics, University of St Andrews, St Andrews, United Kingdom

We derive weak turbulence and quasilinear models for relativistic charged particle dynamics in pitch-angle and energy space, due to interactions with electromagnetic waves propagating (anti-)parallel to a uniform background magnetic field. We use a Markovian approach that starts from the consideration of single particle motion in a prescribed electromagnetic field. This Markovian approach has a number of benefits, including: 1) the evident self-consistent relationship between a more general weak turbulence theory and the standard resonant diffusion quasilinear theory (as is commonly used in e.g. radiation belt and solar wind modeling); 2) the general nature of the Fokker-Planck equation that can be derived without any prior assumptions regarding its form; 3) the clear dependence of the form of the Fokker-Planck equation and the transport coefficients on given specific timescales.

Of particular interest is the ability to derive the governing Fokker-Planck equation, and the required 1st-order (drift) and 2nd-order (diffusion) transport coefficients. Furthermore this method allows for both the inclusion of non-resonant effects, as well as interactions over very short timescales, below a typical de-correlation timescale. All of these nonlinear features have recently been shown to be important for high-energy electron dynamics, and are beyond the reach of the standard resonant diffusion quasilinear theory.

This concise derivation and discussion of the weak turbulence and quasilinear theories using the Markovian framework is physically very instructive and may form fundamental groundwork for future studies.