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## Supervised machine learning to estimate instabilities in chaotic systems: computation of local Lyapunov exponents

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Weather and climate are well known exemplars of chaotic systems exhibiting extreme sensitivity to initial conditions. Initial condition errors are subject to exponential growth on average, but the rate and the characteristic of such growth is highly state dependent. In an ideal setting where the degree of predictability of the system is known in real-time, it may be possible and beneficial to take adaptive measures. For instance a local decrease of predictability may be counteracted by increasing the time- or space-resolution of the model computation or the ensemble size in the context of ensemble-based data assimilation or probabilistic forecasting.

Local Lyapunov exponents (LLEs) describe growth rates along a finite-time section of a system trajectory. This makes the LLEs the ideal quantities to measure the local degree of predictability, yet a main bottleneck for their real-time use in operational scenarios is the huge computational cost. Calculating LLEs involves computing a long trajectory of the system, propagating perturbations with the tangent linear model, and repeatedly orthogonalising them. We investigate if machine learning (ML) methods can estimate the LLEs based only on information from the system's solution, thus avoiding the need to evolve perturbations via the tangent linear model. We test the ability of four algorithms (regression tree, multilayer perceptron, convolutional neural network and long short-term memory network) to perform this task in two prototypical low dimensional chaotic dynamical systems. Our results suggest that the accuracy of the ML predictions is highly dependent upon the nature of the distribution of the LLE values in phase space: large prediction errors occur in regions of the attractor where the LLE values are highly non-smooth. In line with classical dynamical systems studies, the neutral LLE is more difficult to predict. We show that a comparatively simple regression tree can achieve performance that is similar to sophisticated neural networks, and that the success of ML strategies for exploiting the temporal structure of data depends on the system dynamics.