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Koopman eigenfunctions estimation from reproducing kernel Hilbert space manifold, and ensemble data assimilation

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This study aims at proposing a new framework to perform ensemble-based estimations of dynamical trajectories of a geophysical fluid flow system. To perform efficient estimations, the ensemble members are embedded in a set of evolving reproducing kernel Hilbert spaces (RKHS) defining a manifold of spaces, we nicknamed Wonderland, due to its analytical properties.

The method proposed here is designed to deal with very large scale systems such as oceanic or meteorological flows, where it is out of the question to explore the whole attractor, neither to run very long time simulations. Instead, we propose to learn the system locally, in phase space, from an ensemble of trajectories.

The novelty of the present work relies on the fact that the feature maps between the native space and the RKHS manifold are transported by the dynamical system. This creates, at any time, an isometry between the tangent RKHS at time t and the initial conditions. This has several important consequences. First, the kernel evaluations are constant along trajectories, instead to be attached to a system state. By doing so, a new ensemble member embedded in the RKHS manifold at the initial time can be very simply estimated at a further time. This framework displays striking properties. The Koopman and Perron-Frobenius operators on such RKHS manifold are unitary, even though the system might be non invertible. They are furthermore uniformly continuous (with bounded generators) and diagonalizable. As such they can be rigourously expended in exponential forms.

This set of analytical properties enables us to provide a practical estimation of the Koopman eigenfunctions. In the proposed strategy, evaluations of these Koopman eigenfunctions at the ensemble members are exact. To perform robust estimations, the finite-time Lyapunov exponents associated with each Koopman eigenfunction (which are easily accessible on the RKHS manifold as well) are determined. On this basis, we are able to filter the kernel by removing contributions of the Koopman modes that exceed the predictability time. We show that it leads to robust estimations of new unknown trajectories. This framework allows us to write an ensemble-based data assimilation problem, where constant-in-time linear combinations coefficients between ensemble members are sought in order to estimate the QG flow based on noisy swath observations.

The methodology is demonstrated on a barotropic quasi-geostrophic model of a double gyres.

After comparing various kernels and provided guidelines to adapt the kernel with the spread of the ensemble, we show isometry and Koopman-filtered reconstructions. Finally, the data assimilation is presented.