Analysis and numerical experiments on extreme waves through oblique interaction of solitary waves

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Extreme waves can randomly arise in crossing seas with waves coming from two or more main directions. Linear superposition with weak nonlinearity has been proposed to explain "every-day" extreme waves, i.e. waves with an amplitude of at least twice the amplitude of those in the ambient sea. Alternatively, higher-order nonlinear effects in statistical distributions have been simulated and observed to lead to extreme waves. In the former case, it has been proposed to reserve the term "rogue waves" for very high and steep waves. Hence, we have investigated exact and numerical "rogue-wave" solutions of water-wave equations for crossing seas but in a deterministic manner. Two exact web-solitons have been analysed for the unidirectional Kadomtsev-Petviashvili equation (KPE) and numerical solutions have been simulated for the bi-directional, higher-order Benney-Luke equations (BLE) in two horizontal dimensions, the latter seeded at an initial time by either one of these two exact solutions of the KPE. The first exact solution of the KPE is well-known and consists of two main soliton branches of amplitude A, interacting under an angle, to lead to a branch with amplitude 4A. The second exact solution of the KPE is less well-known and consists of three main soliton branches, each with far-field amplitude A, involving waves coming from three directions, and it leads at one point in space and time to an extreme-wave splash. We analyse this exact three-line soliton solution under a symmetric set-up and show in a novel analysis that its maximum, limiting amplification is 9A for a certain angle between the main solitary wave travelling in the positive x-direction and the two other symmetrically-aligned solitary waves. Due to a phase shift, it is only possible to reach the ninefold amplification asymptotically, given a suitable small parameter.

Simulation of these solutions is cumbersome given that they travel fast and exist on an infinite horizontal plane. Given the symmetry in both solutions, we artificially place them as initial condition on a sufficiently-large domain periodic in the x,y-directions, and show, by using further (approximate) symmetry around a y-level, that half a domain suffices with two solid walls and periodicity in the x-direction. Effectively, we have thus created cnoidal-wave solutions of crossing seas. Hence, we seed simulations of the BLE with the exact solutions at some initial time and use geometric or variational (finite-element) integrators to discretise the BLE in space and time, thus preserving phase-space volume, mass and keeping energy oscillations small and bounded, a methodology geared to avoid artificial numerical damping of wave amplitudes. Simulations at different resolutions (using polynomial or p-refinement) reveal that these two types of extreme
waves or web-solitons reach maximum amplifications of circa 3.65 to 4.0 as well as circa 7.8, respectively, for maximum amplifications of 4.0 and circa 8.4 in the exact KPE solutions. Deviations of the exact solutions emerge also because of minor secondary waves created after the initial time, which cause the far-field soliton(s) of original amplitude \( A \) to oscillate in amplitude a bit, diffusing the definition of what the maximum amplification is.