



Satellite and terrestrial spherical harmonic coefficients of the external gravitational potential do not match

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In geodesy, the Earth's external gravitational field is often modelled by a single finite set of external spherical harmonic coefficients. The coefficients are usually derived from satellite or terrestrial gravitational data or by a suitable combination of both. It is known, however, from previous theoretical studies that satellite and terrestrial coefficients are conceptually different and, in principle, do not match. On the one hand, both types can describe the external potential with an arbitrary accuracy in the space above the limit sphere encompassing the gravitating body. On the other hand, when it comes to realistic bodies, only terrestrial coefficients can achieve the same in the space that is below the limit sphere but external to the gravitating body. The price paid to achieve the latter is the fact that the terrestrial coefficients no longer match the satellite coefficients, introducing a conceptual coefficients inconsistency. Using a carefully designed simulated yet realistic closed-loop environment, we numerically reveal in this contribution the different nature of the two coefficients sets. Taking the irregularly-shaped asteroid (101955) Bennu as the gravitating body, we show that, unsurprisingly, the satellite coefficients indeed lead to an excellent accuracy outside the limit sphere (relative accuracy of 10^{-14} in double precision) but produce grossly invalid results below the limit sphere due to the divergence of spherical harmonics. After this exercise, the real challenge of the study was to reliably compute terrestrial coefficients for as complex body as the asteroid Bennu. After computations that took altogether 90 CPU years, we were able to scrutinize the terrestrial coefficients with the relative accuracy of 10^{-6} on the surface of Bennu, that is, below the limit sphere. The results clearly demonstrate the different nature of the two coefficients sets. For instance, it is evident that, from the theoretical point of view, it is rather dangerous to evaluate partial sums from terrestrial coefficients on the Earth's surface. Furthermore, some of the near-surface applications of terrestrial coefficients (e.g., quasigeoid-to-geoid separation or residual terrain modelling) become questionable. As a consolation, the accuracy and the resolution of our Earth's gravitational field models are currently so poor (or excellent, depending on the context) that it will probably take some time for us to encounter these effects with real-world Earth's gravitational data.