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Ensemble forecasts in reproducing kernel Hilbert space family: Application on a multilayer quasi-geostrophic numerical simulation

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This study aims at proposing a new framework to perform ensemble-based estimations of dynamical trajectories of a geophysical fluid flow system. To perform efficient estimations, the ensemble members are embedded in a set of evolving reproducing kernel Hilbert spaces (RKHS) defining a family of spaces.

The method proposed here is designed to deal with very large scale systems such as oceanic or meteorological flows, where it is out of the question to explore the whole attractor, neither to run very long time simulations. Instead, we propose to learn the system locally, in phase space, from an ensemble of trajectories.

The novelty of the present work relies on the fact that the feature maps between the native space and the RKHS manifold are transported by the dynamical system. This creates, at any time, an isometry between the tangent RKHS at time t and the initial conditions. This has several important consequences. First, the kernel evaluations are constant along trajectories, instead to be attached to a system state. By doing so, a new ensemble member embedded in the RKHS manifold at the initial time can be very simply estimated at a further time. This framework displays striking properties. The Koopman and Perron-Frobenius operators on such RKHS manifold are unitary, uniformly continuous (with bounded generators) and diagonalizable. As such they can be rigorously expended in exponential forms.

This set of analytical properties enables us to provide a practical estimation of the Koopman eigenfunctions. In the proposed strategy, evaluations of these Koopman eigenfunctions at the ensemble members are exact. To perform robust estimations, the finite-time Lyapunov exponents associated with each Koopman eigenfunction (which are easily accessible on the RKHS manifold as well) are determined. On this basis, we are able to filter the kernel by removing contributions of the Koopman modes that exceed the predictability time. We show that it leads to robust estimations of new unknown trajectories. This framework allows us to write an ensemble-based data assimilation problem, where constant-in-time linear combinations coefficients between ensemble members are sought in order to estimate the QG flow based on noisy swath observations.

The methodology is demonstrated on a multilayer quasi-geostrophic model representative of the Gulf Stream area in the North Atlantic at a 10 km resolution and considering 100 training

ensemble members. We show the ability of the method to estimate trajectories knowing the initial condition of a new ensemble member. Moreover, ensemble-based data assimilation is performed based on realistic swaths of altimetry observations.