



The Challenge of Non-Markovian Energy Balance Models in Climate

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Hasselmann’s paradigm, introduced in 1976 and recently honoured with the Nobel Prize, can, like many key innovations in the sciences of climate and complexity, be understood on several different levels, both technical and conceptual. It can be seen as a mathematical technique to add stochastic variability into pioneering energy balance models (EBMs) of Budyko and Sellers. On a more conceptual level, it used the mathematics of Brownian motion to provide an abstract superstructure linking slow climate variability to fast weather fluctuations, in a context broader than EBMs, leading Hasselmann to posit the need for negative feedback in climate modelling.

Hasselmann’s paradigm itself has much still to offer us [e.g. Calel et al, *Nature Communications*, 2020], but naturally, since the 1970s a number of newer developments have built on his pioneering ideas. One important one has been the development of a rigorous mathematical hierarchy that embeds Hasselmann-type models in the more comprehensive Mori-Zwanzig (MZ) framework (e.g. Lucarini and Chekroun, *Nature Reviews Physics*, 2023). Another has been the interest in long range memory in stochastic EBMs, notably Lovejoy et al’s Fractional Energy Balance Equation [FEBE, discussed in this week’s Short Course SC5.15]. These have a memory with slower decay and thus longer range than the exponential form seen in Hasselmann’s EBM. My presentation [based on Watkins et al, in review at *Chaos*] attempts to build a bridge between MZ-based extensions of Hasselmann, and the fractional derivative-based FEBE model. I will argue that the Mori-Kubo overdamped Generalised Langevin Equation, as widely used in statistical mechanics, suggests the form of a relatively simple stochastic EBM with memory for the global temperature anomaly, and will discuss how this relates to FEBE.

