



The selective boundary crossing in stochastic processes driven by white shot noise

Giulio Calvani and Paolo Perona

PL-LCH, EPFL, Lausanne, Switzerland (giulio.calvani@epfl.ch)

Many physical, chemical, financial, and ecological processes show the presence of a threshold, which may affect their dynamics. For instance, chemical reactions occur when the energy reaches the activation value; for insurance companies and banks, ruin may happen when the balance drops down a minimum amount; in fluvial hydraulics, sediment transport, and morphodynamic processes start when bed shear stresses (i.e., flow discharge) overcome a critical threshold. Other processes may show the presence of additional boundaries. For stochastic processes, whose dynamics depends on the frequency and magnitude of random fluctuations, it is interesting to know the average time the process takes to reach one of the critical boundaries starting from a known value. This quantity is known in the literature as *Mean First Passage Times* (MFPTs). The quantification of the MFPTs is usually performed by considering one threshold, only. When two or multiple thresholds are present, one may consider the MFPTs of reaching either one of the thresholds, without having passed the other ones. Such a selective condition is referred to in the literature as *splitting probability*. In this work, we consider stochastic processes governed by a typical Langevin equation with deterministic drift and random instantaneous jumps (white shot noise). We perform a statistical-trajectory-analysis starting from a point between two thresholds and derive exact relationships of the splitting probabilities and the MFPTs of crossing one threshold, only, based on process-dependent dimensionless parameters. Such formulations are then explicitly given for the cases of constant and linear drift functions and both positive and negative jumps. We test the derived formulations against data from MonteCarlo simulations, by varying the process parameters, the starting point, and the values of the thresholds. The comparison shows very good agreement and confirms the correctness of the derived relationships. Additionally, the analysis highlights the role played by the dimensionless parameters. Then, data from flow measurements in a river are considered and we successfully test the formulations against the duration of the raising limb of high-stage events. The derived formulations can be readily applied to calculate both the duration of the raising and the falling limbs of flow events, which are important quantities for engineering applications, as well as for modeling purposes in river eco-morphodynamics.