Continuous-time state-space time series models for delta-O-18 and delta-C-13

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Time series analysis of delta-O-18 and delta-C-13 measurements from benthic foraminifera for purposes of paleoclimatology is challenging. The time series reach back tens of millions of years, they are relatively sparse in the early record and relatively dense in the later, the time stamps of the observations are not evenly spaced, and there are instances of multiple different observations at the same time stamp (Westerhold et al., 2020, Science 369 p. 1383). The time series appear non-stationary over most of the historical record with clearly visible temporary trends of varying directions. In this paper, we propose a continuous-time state-space framework to analyze the time series prepared in Westerhold et al. (2020). State space models are uniquely suited for this purpose, since they can accommodate all the challenging features mentioned above. We specify univariate models and joint bivariate models for the two time series of delta-O-18 and delta-C-13. The models are estimated using maximum likelihood by way of the Kalman filter recursions. The suite of models we consider has an interpretation as an application of the Butterworth filter (Gomez 2001 [JBES 19 p. 365], Harvey & Trimbur 2003 [REStat 85 p. 244]). We propose model specifications that take the origin of the data from different studies into account and that allow for a partition of the total period into sub-periods following Westerhold et al. 2020, which we have been able to confirm with a statistical method (Larsen et al. 2024: Estimating Breakpoints between Climate States in Paleoclimate Data, abstract submitted to EGU General Assembly Session CL3.2.3). The models can be used, for example, to generate evenly time-stamped data by way of Kalman filtering. They can also be used, in future work, to analyze the relation to proxies for CO2 concentrations.