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Accurate solutions of highly oscillatory systems under large time steps using higher-order phase averages

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We introduce a time integrator for a prototype model of highly oscillatory PDEs that exhibits accurate solutions even under the usage of large time step sizes. To achieve large time steps, we apply a phase averaging technique that smooths out the fast waves from the system. To avoid the errors that such smoothing usually entails, we use a higher order (HO) phase averaging algorithm based on the idea of [1]. This algorithm expresses the sensitivity of the solutions on the phases in terms of an HO basis which the equations are projected onto. The resulting HO phase corrections reduce the errors in the solutions even for finite averaging windows. Rather than using monomials as such HO basis as originally suggested in [1], here we introduce an alternative basis in terms of exponentials and we discuss its properties.

Similarly to [1], we test this idea on an ODE describing the dynamics of a swinging spring, a model due to Peter Lynch. Although idealized, this model shows an interesting analogy to geophysical flows as it exhibits a high sensitivity of small scale oscillation on the large scale dynamics. On this example, we illustrate that the HO phase averaging method with an exponential basis allows for highly accurate solutions even when using large averaging windows and hence larger time step sizes than standard methods. These HO phase corrections (the reason for this improvement) can be evaluated independently, hence computed in parallel. In contrast, in standard averaging approaches, e.g. [2], arbitrarily accurate solutions could only be obtained with small averaging windows as well as small time step sizes. We present these highly promising results and discuss challenges in generalizing them to highly oscillatory PDEs for applications in simulations of weather, ocean, and climate.

References

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