

Green's function based approach for the light scattering calculation by inhomogeneous particles

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Green's function method is known to be applied for spherically symmetric inhomogeneous particles [1]. In this work we develop a new approach for the light scattering calculation by spherical inhomogeneous particles provided that an inhomogeneity has not necessarily spherical symmetry. The new technique relies upon the Generalized Source Method developed previously for efficient grating diffraction simulation [2]. Rationale of the method rests upon the integral solution of the time-harmonic Helmholtz equation on unknown electric field $\mathbf{E}(\mathbf{r})$:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{inc}(\mathbf{r}) + i\omega\mu_0 \int \mathbf{G}(k_b|\mathbf{r} - \mathbf{r}'|)\mathbf{J}(\mathbf{r}')d\mathbf{r}' \quad (1)$$

Here $\mathbf{E}^{inc}(\mathbf{r})$ is a known incidence field, $k_b^2 = \omega^2\varepsilon_b\mu_0$, ε_b is some constant “basis” permittivity, and

$$\mathbf{G}(k_b|\mathbf{r} - \mathbf{r}'|) = -\frac{\hat{e}_r\hat{e}_r}{k_b^2}\delta(\mathbf{r} - \mathbf{r}') + \frac{ik_b}{4\pi} \sum_{nm} C_{nm} \left[\mathbf{M}_{nm}(k_b\mathbf{r}_<) \mathbf{M}_{nm}^{(1)}(k_b\mathbf{r}_>) + \mathbf{N}_{nm}(k_b\mathbf{r}_<) \mathbf{N}_{nm}^{(1)}(k_b\mathbf{r}_>) \right] \quad (2)$$

is the well-known free-space dyadic Green's function eigen decomposition [3]. Notably, it is essentially to keep singular source term in Eq.(2) as our treatment deals with spatially continuous “generalized” sources produced by dielectric permittivity inhomogeneities in form

$$\mathbf{J}_{gen}(\mathbf{r}) = -i\omega [\varepsilon(\mathbf{r}) - \varepsilon_b] \mathbf{E}(\mathbf{r}) \quad (3)$$

Source (3) being substituted in Eq.(1) brings self-consistent integral equation for unknown field.

Discretization of Eq.(1) with source (3) is performed over the vector spherical function basis set and by subdivision of a spherical layer containing inhomogeneities into a set of spherical shells (in order to replace all integrals over radial variable r with finite sums). In each shell with radius r_s permittivity is treated as a function of spherical angles and is decomposed into spherical harmonics $\Delta\varepsilon(\mathbf{r}) = \sum_{nm} [\Delta\varepsilon(r_s)]_{nm} Y_{nm}(\theta, \phi)$. This brings a linear algebraic equation system on unknown field decomposition coefficient vector \mathbf{a} in form

$$(I - RS)\mathbf{a} = \mathbf{a}^{inc} \quad (4)$$

where I is the identity matrix, matrix S “physically” describes scattering in an infinitely thin spherical shell, and matrix R “redistributes” scattered spherical waves between shells in electromagnetically rigorous manner.

The method initially does not contain any assumption on inhomogeneity scale and permittivity contrast, so these parameters affect primarily the number of spherical waves and spherical shells needed to obtain a given solution accuracy. In this first demonstration of the method we concentrate on dielectric spherical particles with spatially continuous inhomogeneity leaving questions of the method application to non-spherical particles for further consideration.

[1] L.-W. Li, P.-S. Kooi, M.-S. Leong, T.-S. Yeo. Electromagnetic dyadic Green's function in spherically multilayered media. IEEE Trans. Microwave Theory. 42:2302-2310 (1994).

[2] A.A. Shcherbakov, A.V. Tishchenko. New fast and memory-sparing method for rigorous electromagnetic analysis of 2D periodic dielectric structures, JQSRT 113:158-71 (2012).

[3] P.M. Morse, H. Feshbach, Methods of Theoretical Physics (McGraw-Hill, New York 1953)