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J. Lock
United States (j.lock@csuohio.edu)

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James A. Lock

Physics Dept., Cleveland State University, Cleveland OH 44115 USA

j.lock@csuohio.edu

Abstract

For scattering of an incident plane wave or shaped beam by a homogeneous spherical particle, the Debye series describes the multiple scattering interaction of the incident beam with the particle surface. Each Lorenz-Mie partial wave scattering amplitude a_n and b_n may be exactly rewritten as an infinite series, whose terms correspond to diffraction, external reflection, transmission, and transmission following any number of internal reflections in the short wavelength limit. Symbolically, the Debye series expansion of the partial wave scattering amplitudes is

$$a_n, b_n = (1/2)\left[1 - r_n^{212} - \sum_{p=1}^{\infty} t_n^{21} (r_n^{121})^{p-1} t_n^{12}\right], \quad (1)$$

where the sphere interior is region 1 and the exterior medium is region 2. The amplitude of a partial wave transmitted into or out of the particle at its surface is t_n^{21} or t_n^{12}, and the amplitude externally or internally reflected at the surface is r_n^{212} or r_n^{121}. These are known as single-interface amplitudes because they describe the interaction of the incident wave at the sphere surface alone.

For scattering of an incident beam by a spherical particle having M concentric layers, each partial wave produces both a transmitted and reflected component at each of the interfaces. This leads to an infinite number of complicated back-and-forth multiple scattering paths as the partial wave eventually progresses through the multi-layer sphere. This complexity, however, can be organized in a physically and mathematically pleasing way. One can define multiple-scattering amplitudes T_n starting from the external medium and ending in the core, or starting from the core and ending in the external medium, that sum all possible sequences of interactions of the partial wave with all the intervening interfaces. One can define similar multiple-scattering reflection amplitudes R_n. There are recursion relations for re-expressing the multiple-interface amplitudes in terms of progressively more complicated combinations of fewer-interface amplitudes, and finally single-interface amplitudes. The partial wave amplitudes for scattering by the composite sphere are then found to have the form

$$a_n, b_n = (1/2)\left[1 - R_n^{M+1,M+1} - \sum_{p=1}^{\infty} T_n^{M+1,1} (R_n^{1,1})^{p-1} T_n^{1,M+1}\right]. \quad (2)$$

The pattern observed in Eq.(1) for a homogeneous sphere continues exactly the same way in Eq.(2) for scattering by a multi-layer sphere when the single-interface amplitudes are replaced by the corresponding multiple-scattering amplitudes.