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Understanding the polarization effects of Jones-Mueller matrices in terms of hyperbolic geometry

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The transversal polarization of a totally polarized plane wave is usually described in terms of a complex 2-component Jones vector. The polarization altering properties of a slab of matter, interacting with such a wave in a linear way, is represented by a 2x2 complex Jones matrix.

More generally, the transversal polarization of a plane wave of natural (or partially polarized) light is described in terms of a real 4-component Stokes vector. The polarization altering properties of a slab of matter is now encoded in a 4x4 real Mueller matrix. An important subset of the set of Mueller matrices is the set of Jones-Mueller matrices, which are Mueller matrices having a corresponding Jones matrix.

The majority of the optical components, used in polarization optics, can be described by a Jones-Mueller matrix. Also, the Rayleigh scattering matrix, used in the vectorial radiative transfer model, is a Jones-Mueller matrix and all the aerosol scattering matrices, measured so far, can be represented as a convex sum of Jones-Mueller matrices.

Stokes vectors are mathematically similar to the four-momentum vectors in Special Relativity. Consequently, Stokes vectors naturally possess a hyperbolic (or Lorentzian) geometry. The aim of this talk is to show that the polarization effects of a Jones-Mueller matrix can be most easily understood in terms of this geometry. Furthermore, the mathematical formulation of how a Stokes vector is transformed by a Jones-Mueller matrix can be most simply expressed by using this hyperbolic geometry.

In particular, it will be shown that the action of a particular Jones-Mueller matrix, called a diattenuator, on a Stokes vector can be understood in terms of the addition law for relativistic velocities from Special Relativity. An important simplification in the resulting mathematical expressions further arises if the degree of polarization of a Stokes vector is represented by a hyperbolic polarization angle. Then, the output hyperbolic polarization angle is related to the diattenuator hyperbolic polarization angle and the input hyperbolic polarization angle by the hyperbolic law of cosines holding in a hyperbolic triangle.