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Conditioning of the weak-constraint 4DVAR problem

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The aim of data assimilation is to provide a statistically optimal estimate for the state of a system described by a dynamical model. In the context of Numerical Weather Prediction, variational techniques, more specifically, 4-dimensional variational data assimilation (4DVAR) algorithms are now at the epicentre of the computational machinery driving weather forecasts. 4DVAR aims to assimilate observations through the minimisation of a cost function with the constraint of the model. The key assumption is that the evolution of the state is exactly described by the dynamical model, but in practice this is often not true. Relaxing this assumption to allow for errors in the model gives rise to a more general form of 4DVAR: weak-constraint 4DVAR.

Considering the assimilation problem where the accuracy of the model is in question leads to different minimisation problem with more degrees of freedom. We present two formulations of the problem, which are now receiving increasing research focus in the Numerical Weather Prediction community. The conventional use of gradient-based minimisation techniques puts emphasis on the importance of the Hessian of the cost function. The condition number of the Hessian is indicative of the accuracy of the solution and the rate of convergence of the iteration process used to solve the problem.

Theoretical bounds on the condition number of the Hessian are derived and demonstrated on a simple one-dimensional advection model with periodic domain. We also present numerical results to show the sensitivity of the problem to length-scales, variances and assimilation window arrangement. Finally, we show some fundamental differences in both formulations when applied to the advection model.