



## Some reasoning behind self-similarity of stratified turbulence

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Since Richardson (1920), it is generally agreed that the effect of stratification on the shear-generated geophysical turbulence is determined by the gradient Richardson number  $Ri = (N/S)^2$ , where  $N$  is the Brunt-Vaisala frequency,  $S = dU/dz$  is vertical shear of the mean wind/current velocity  $U$ , and  $z$  is vertical coordinate. The concept of Richardson-number similarity postulates that dimensionless characteristics of turbulence are universal functions of  $Ri$ .

Monin and Obukhov (1954) have proposed for the atmospheric surface layer a widely recognised *Monin-Obukhov similarity theory* (MOST) postulating that dimensionless characteristics of turbulence are fully determined by the ratio  $z/L$ , where  $L = -(u_*)^3/F_b$  is the Obukhov length scale,  $u_*$  is friction velocity and  $F_b$  is vertical turbulent fluxes of buoyancy. Nieuwstadt (1984) employed local,  $z$ -dependent values of  $F_b$  and  $u_*$  instead of the surface values, and demonstrated applicability of such version of MOST to almost entire steady-state stable boundary layer (except its upper part influenced by interactions between the boundary layer and free flow). The  $Ri$ -similarity is consistent with MOST: in the surface layer  $Ri$  is a monotonously increasing function of  $z/L$  and vice versa (e.g., Sorbjan, 2010). However, the  $Ri$ -similarity it is also applicable to any stably stratified flow with locally balanced turbulence. In the essentially unstable stratification, both MOST and  $Ri$ -similarity fail because of development of self-organised structures embracing the entire convective zone and transporting turbulence faraway from its seat of origin (Elperin et al., 2006; Zilitinkevich et al., 2006).

Besides the concept of similarity, MOST includes asymptotic analyses for the strongly stably stratified turbulence, assuming its independence of height ( $z$ -less stratification); and for the strongly unstably stratified turbulence, assuming its independence of friction velocity (free convection). Asymptotic relationships resulted from these analyses have never been questioned, and still form the bases for parameterization of the surface-layer turbulence in modelling applications. We show that both of the above assumptions are inconsistent with observational evidence and modern theory.

In this paper we examine self-similarity of stably stratified turbulence employing the EFB turbulence closure (Zilitinkevich et al, 2013) and available experimental data. It is shown that self-similarity is inherent to any locally balanced, shear-generated turbulence. The primitive similarity criterion is the flux Richardson number  $Ri_f = -F_b/(\tau S)$ . The  $Ri_f$ - and  $Ri$ -similarities are principally equivalent. MOST is justified not only in the surface layer but also in the boundary layer provided that generation of turbulent kinetic energy  $E_K$  is locally balanced by its dissipation and conversion into turbulent potential energy  $E_P$ , whose production is also balanced by dissipation.

Besides these quite expected conclusions, the EFB theory has given an insight into the nature of self-similarity. The fact that dimensionless combinations of different turbulent parameters (energies, fluxes, eddy viscosity  $K_M \equiv \tau/S$ , eddy conductivity  $K_H \equiv F_b/N^2$ , etc.) are universal functions of  $Ri_f$ ,  $Ri$  or  $z/L$  is explained by the equivalence of the dissipation time scales for all second moments under consideration. It has become clear that the concept of self-similarity is generally irrelevant to the third and higher moments. Conclusions from MOST related to the so-called  $z$ -less stratification regime are shown to be partially misleading. In particular, MOST prescribes constancy of the turbulent Prandtl number:  $Pr_T = K_M/K_H = \text{constant}$ , whereas extensive experimental data, as well as the EFB closure, yield strong large- $Ri$  asymptotic dependence:  $Pr_T \approx 4Ri$ .

In the imbalanced turbulence there are no grounds to expect the exact self-similarity. Moreover, traditional similarity criteria, such as  $Ri_f$ ,  $Ri$  or  $z/l$ , based on local parameters, determined in models diagnostically, become strongly variable and unreliable. To overtake this difficulty, we propose a concept of approximate similarity based on the "energy stratification criterion"  $\Pi \equiv E_P/E_K$ . In the steady state,  $\Pi = C_p Ri_f / (1 - Ri_f)$ , where  $C_p$  is empirical constant, so that  $\Pi$ -similarity reduces to traditional  $Ri_f$ - or  $Ri$ -similarities. However, in the imbalanced turbulence where traditional similarity concepts fail, the  $\Pi$ -similarity, based on prognostic, trustworthy values of turbulent energies, remains a reasonable approximation.