

[\[Session UP1.2\]](#)

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The Zilitinkevich Scale in Life and Science

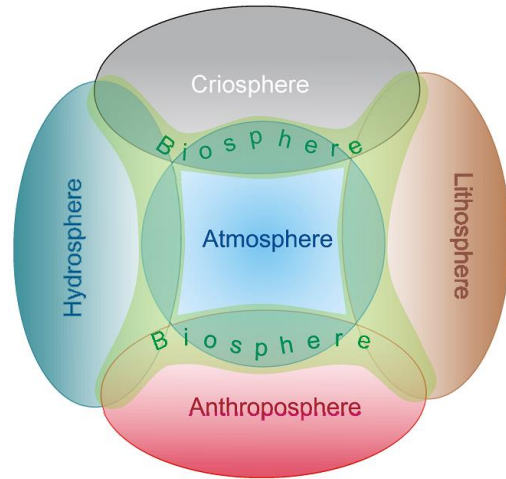
By Igor Esau



The 2015 Alfred Wegener Medal and the 2019 International Meteorological Organization Prize for outstanding contribution to fundamentals of meteorology and climatology



Sergej Zilitinkevich deserves credit for the creation of the fundamentals of the theory of stratified PBLs ... *"In particular, for his rotation-stratification depth-scale for stable PBLs, now referred to as the Zilitinkevich scale, and the PBL bulk resistance and heat-mass-transfer laws expressing the near-surface turbulent fluxes through external parameters"*.



He extended these laws to extreme stratifications accounting for the newly recognised non-local features of stable PBLs and organised structures in convective PBLs. Another important contribution is the prognostic equation for the depth of evolving boundary layers, accounting for the turbulent energy spin-up.

The Zilitinkevich Scale in Life: *From Venus to Prisoners' camps*

1968: the Head of Planetary Atmosphere Research in the USSR Space Research Programme "Venus".

1970: the Chair of the USSR Commission on Air-Sea Interaction.

"The methods of A. S. Monin and S. S. Zilitinkevich were employed to determine momentum and heat fluxes in the boundary layer and surface temperature of the planet, and to make "convective adjustment", ... in the free atmosphere."

Chalikov, D.V., Monin, A.S., Turikov, V.G., and Zilitinkevich, S.S., 1971: Numerical experiments on the general circulation of the Venus atmosphere. *Tellus*, 23, No. 6, 483-488.

"... As usual, I sit on my upper bed in our overcrowded jail cellroom. I tried to work with equations not only for spending my time but also for keeping my mental health in such overwhelming circumstances. ... My such a strange habit got a certain degree of respect from my cellmates. They sometimes lowered their voices chatting to each other: "Professor is thinking!"

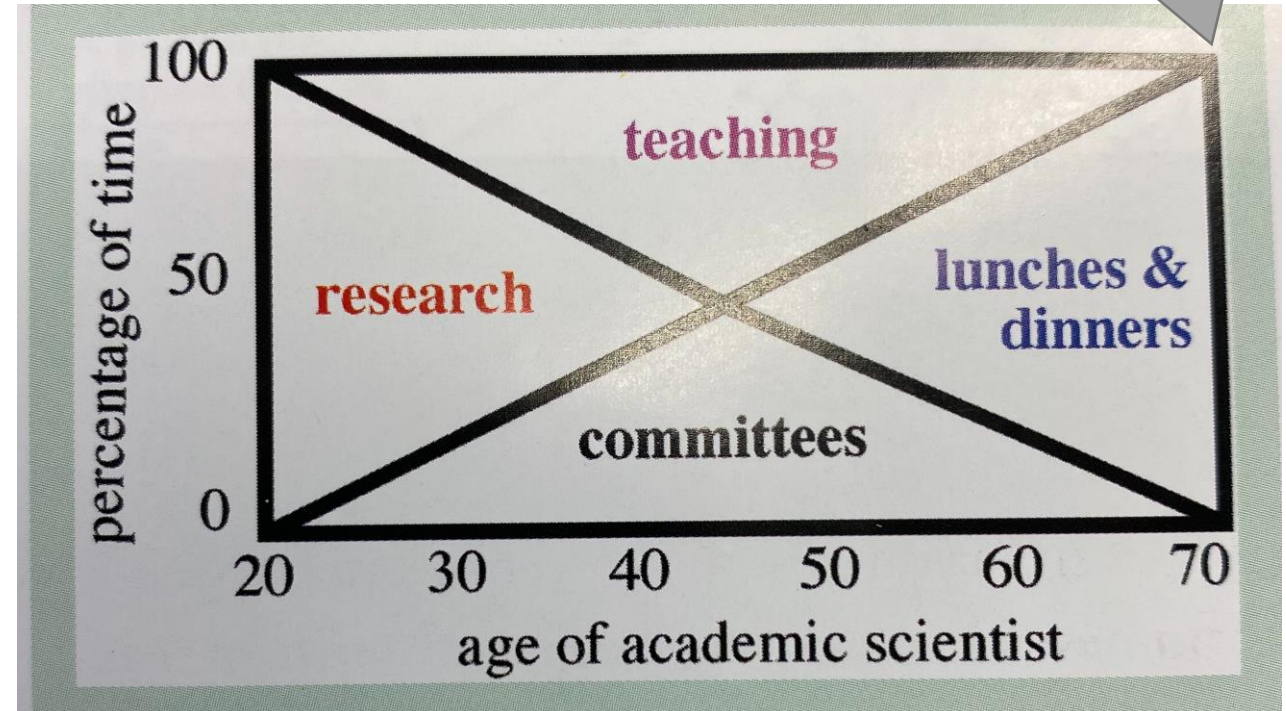
From "Hei, Professor!" – a biographical book by S. Zilitinkevich



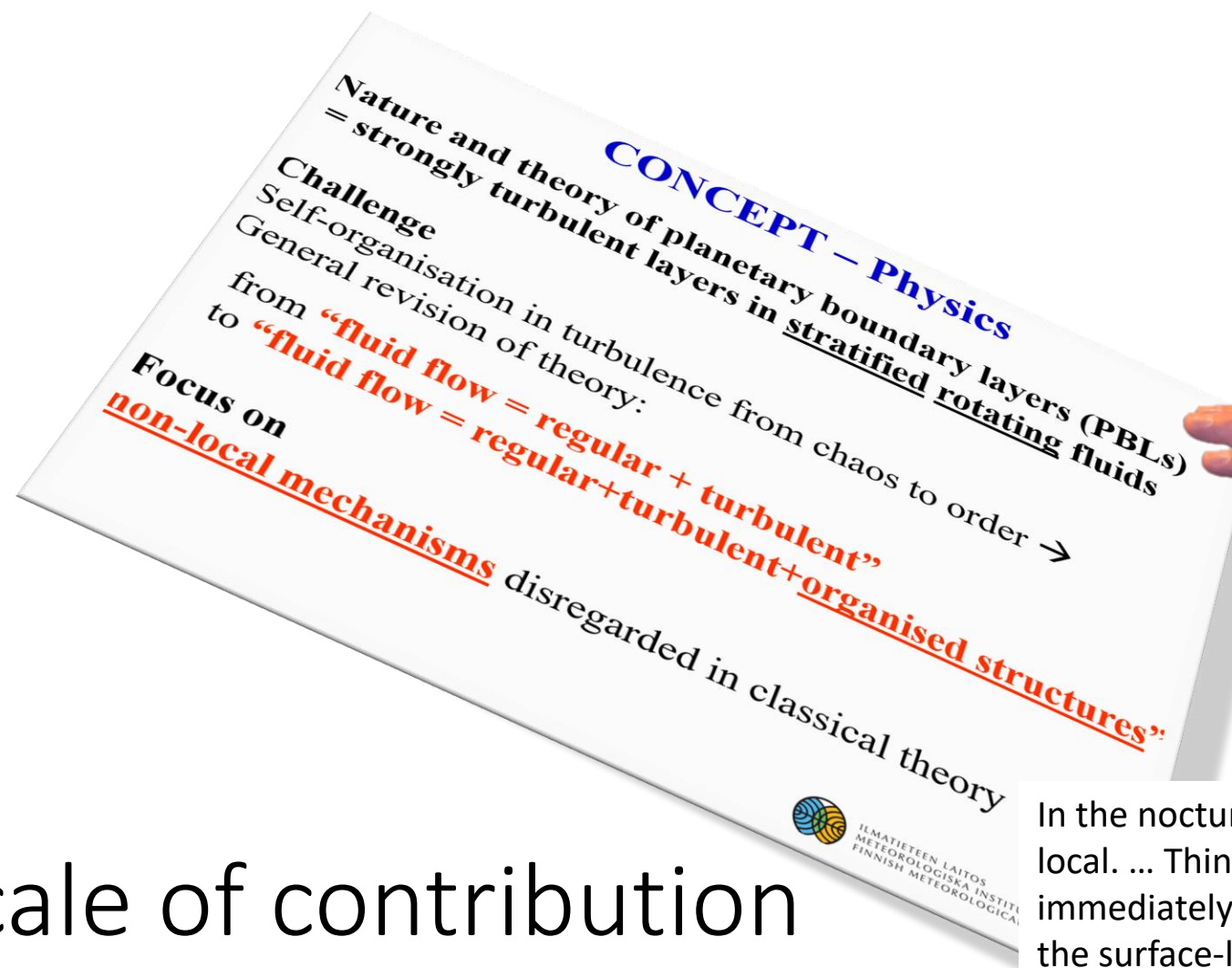
The Zilitinkevich Scale in Science

- Sergej Zilitinkevich was passionate for truly large-scale international science that combined the best of multidisciplinary approaches and multicultural research schools.
- He has been leading international efforts to develop an energetically consistent turbulence theory that could be applied to strongly stratified environmental flows.
- His last project - the Pan-Eurasian experiment (PEEX) - connects more than 100 research groups from 20 countries. It aims to challenges of global warming, atmospheric pollution, biodiversity loss, and energy production recognizing the increasing role of the cold climate areas in the context of global change.

2005 Important discussions that lay out foundations of the Total Turbulence Energy approach



From Steven Benner's book
'Life, the Universe... and the Scientific Method'



The scale of contribution

Zilitinkevich, S.S., 2002. Third-order transport due to internal waves and non-local turbulence in the stably stratified surface layer. Q. J. R. Meteorol. Soc. 128, 913–925. <https://doi.org/10.1256/0035900021643746>

In the nocturnal SBLs the nature of turbulence is basically local. ... Things are different in *long-lived* SBLs situated immediately below the stably stratified free flow. Here, the surface-layer turbulence is affected by the free-flow Brunt–Vaisala frequency, N . **The surface layer ... is separated from the free atmosphere by the upper nine-tenths of the SBL comprising hundreds of metres.** **Traditional concepts fail to explain such distant links. ...**

Stratification, Rotation, Non-locality

Zilitinkevich, S.S., 1972: On the determination of the height of the Ekman boundary layer. Boundary-Layer Meteorol., 3, 141- 145.

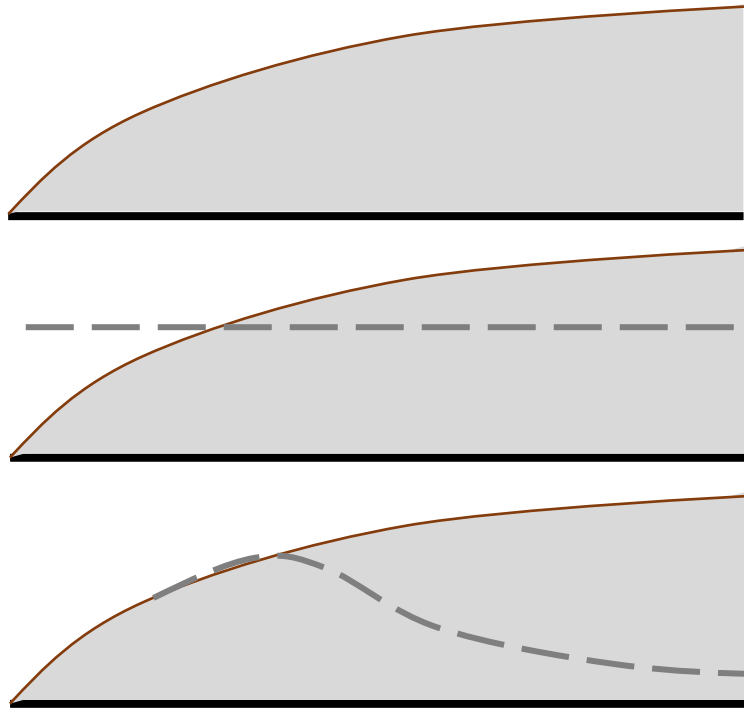
Environmental turbulence differs from turbulence in majority of fluid dynamics applications not only by very high Reynolds number but also by a pervasive role of density stratification and system rotation expressed through:

- Monin-Obukhov length scale
- Coriolis parameter

The Zilitinkevich scale - a length scale of a rotation-stratification turbulent mixing in stably stratified planetary boundary layers.

This scales limits the thickness of the mixed layer (the PBL depth), thus, having significant implications on air quality, heat and moisture transports, and the Earth's climate as the whole.

Stratification, Rotation, Non-locality



L. Prandtl – no rotation, no stratification, no non-locality:

$$h \propto \frac{x}{\sqrt{Re}} \propto \frac{x}{\sqrt{U}} \cdot \left(\frac{\nu}{z_0} \right)^{-1/2} \rightarrow \infty$$

W. Ekman – rotation, no stratification, no non-locality:

$$h \propto \left(\frac{K_M}{|f|} \right)^{\frac{1}{2}} \propto \left(\frac{u_* l}{|f|} \right)^{\frac{1}{2}} = c_{TN} \left(\frac{u_* h}{|f|} \right)^{\frac{1}{2}} = c_{TN} \frac{u_*}{|f|} \rightarrow const$$

S. Kitaigorodskii – no rotation, bottom stratification, no non-locality:

$$h \propto L = \frac{u_*^3}{B_s} \rightarrow const$$

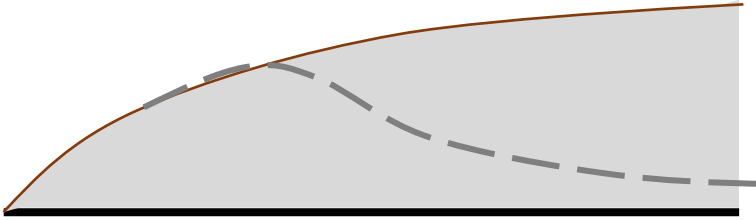
S. Kitaigorodskii and S. Joffre – no rotation, top stratification, non-locality:

$$h \propto \frac{u_*}{N} \rightarrow const$$

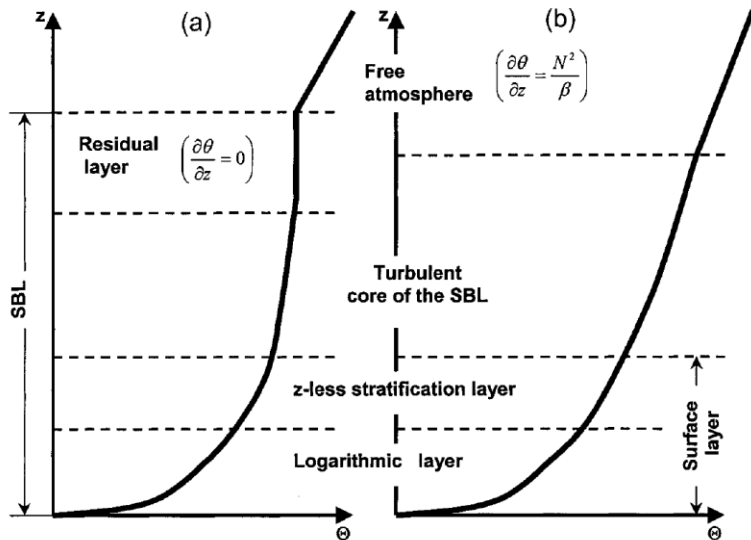
Kitaigorodskii SA 1960. On the computation of the thickness of the wind-mixing layer in the ocean. *Izv. AN SSSR. Ser. Geofiz.* **3**: 425–431.

Kitaigorodskii SA, Joffre SM 1988. In search of simple scaling for the heights of the stratified atmospheric boundary layer. *Tellus* **40A**: 419–433.

Stratification, Rotation, Non-locality



$$\frac{1}{h^2} = \frac{1}{h_E^2} + \frac{1}{h_{NS}^2} + \frac{1}{h_{CN}^2}$$



Construct the boundary layer scaling on basis of the Ekman Layer.

C. Rossy and Montgomery – rotation, no stratification, no non-locality:

$$h_E \propto \left(\frac{K_M}{|f|} \right)^{\frac{1}{2}} \propto \left(\frac{u_* l}{|f|} \right)^{\frac{1}{2}} = c_{TN} \left(\frac{u_* h}{|f|} \right)^{\frac{1}{2}} = c_{TN} \frac{u_*}{|f|} \rightarrow const$$

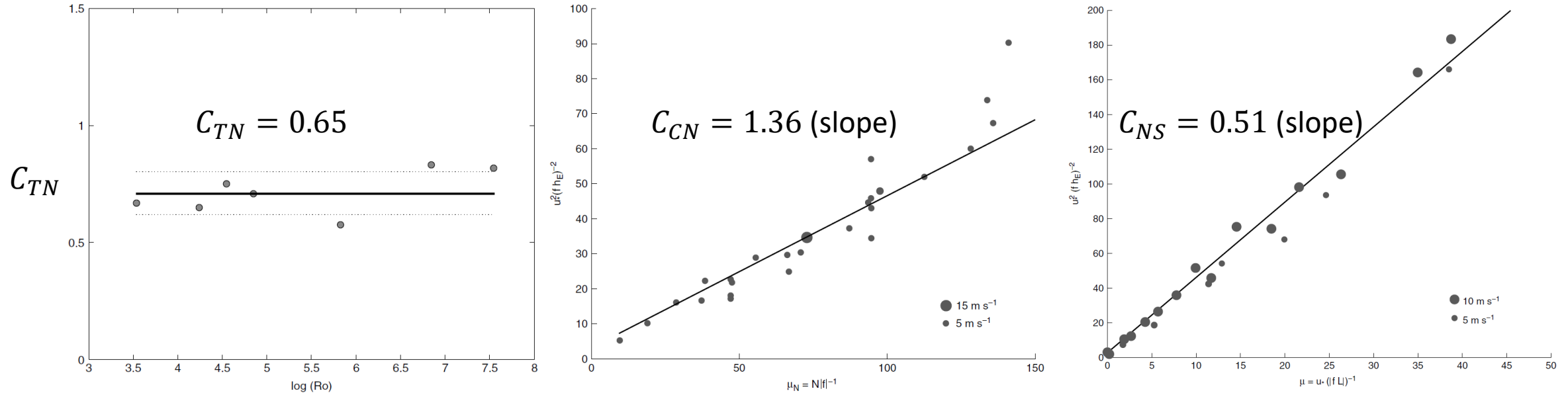
S. Zilitinkevich (1973) – rotation, bottom stratification, no non-locality:

$$h_{NS} \propto h_E \cdot c_{NS} \frac{u_* \sqrt{|f|}}{\sqrt{B_s}} \rightarrow const$$

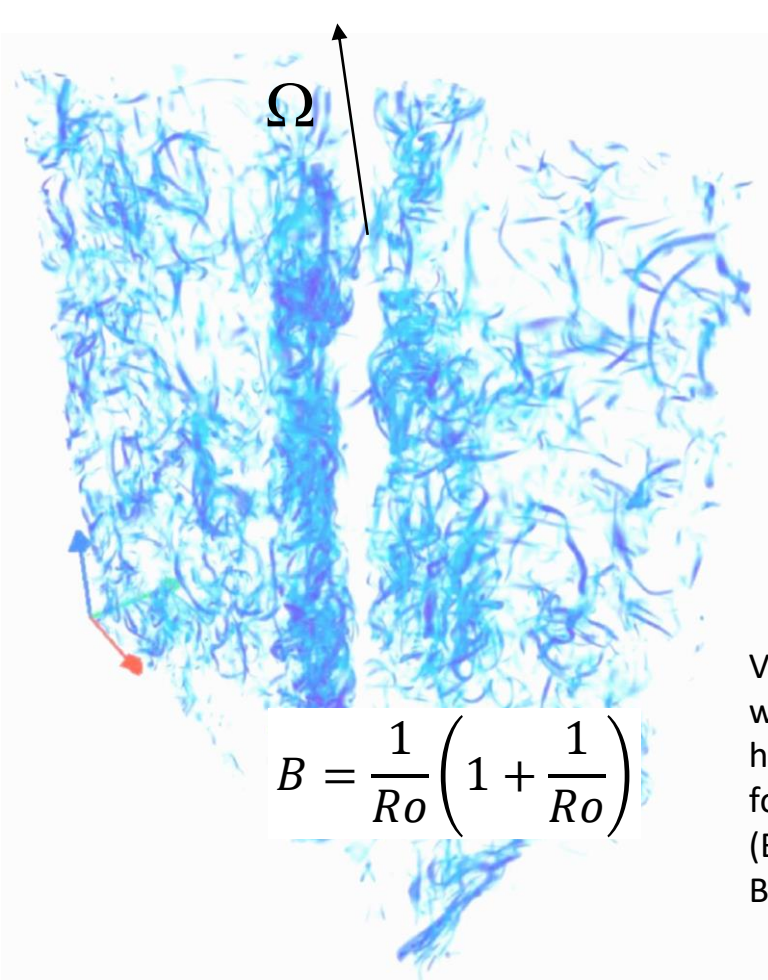
S. Pollard – rotation, top stratification, non-locality:

$$h_{CN} \propto h_E \cdot c_{CN} \frac{\sqrt{|f|}}{\sqrt{N}} \rightarrow const$$

Asymptotic constants



Zilitinkevich S, Esau I, Baklanov A 2007. Further comments on the equilibrium height of neutral and stable planetary boundary layers. *Q. J. R. Meteorol. Soc.* **133**: 265–271.



Rotation: «Coriolis Stratification»

The Coriolis force acts through «Coriolis Stratification» as the Bradshaw frequency could be considered as an analogue of the Brunt-Vaisala frequency in stratified fluid

$$\omega_B = 2\Omega \left(2\Omega - \frac{\partial U}{\partial y} \right)$$

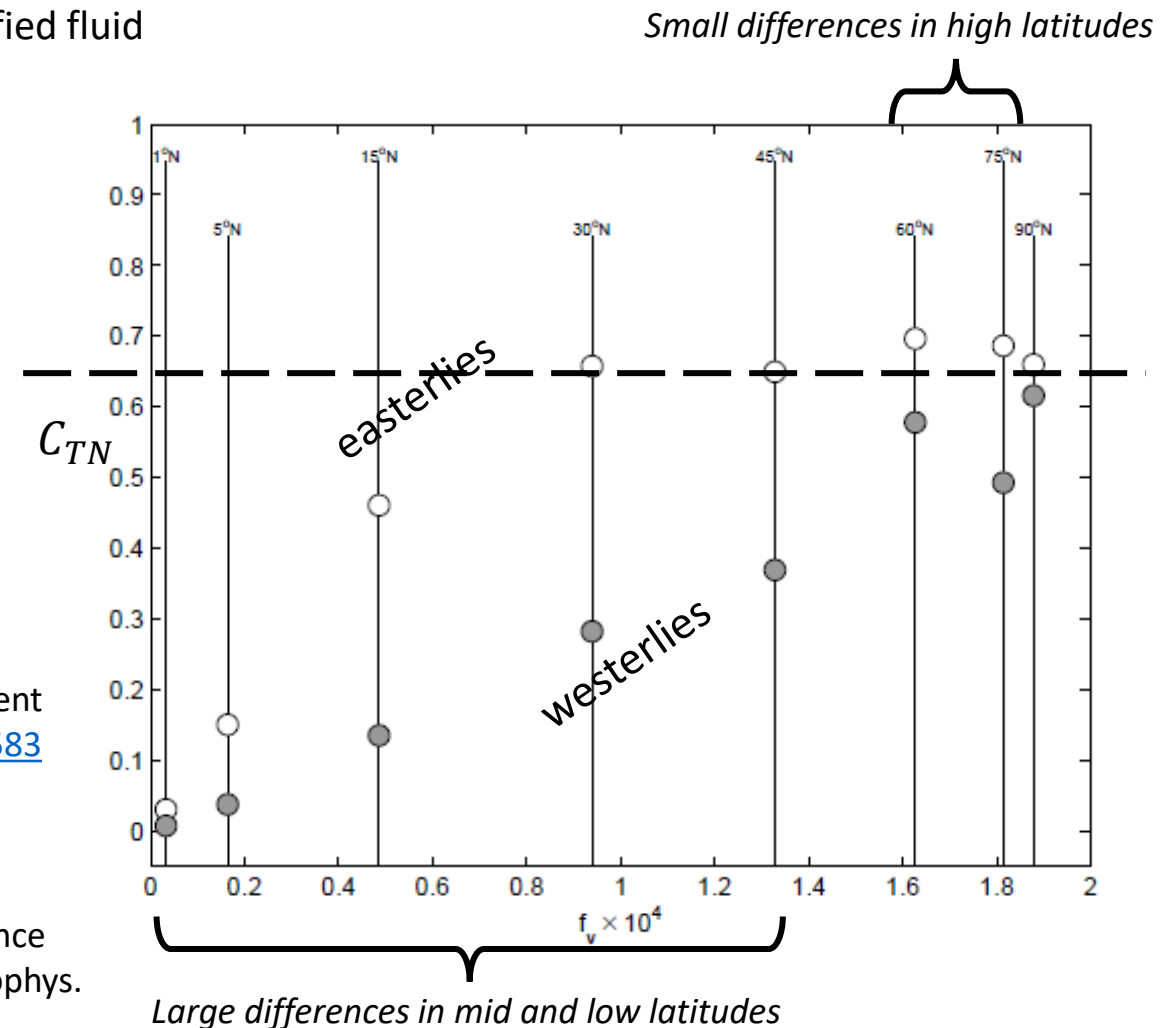
Vorticity makes a difference between westerlies and easterlies scaled by the horizontal component of the Coriolis force, stabilizing ($B > 0$) or destabilizing ($B < 0$) the flow as determined by the Bradshaw-Richadson number

$$B = \frac{1}{Ro} \left(1 + \frac{1}{Ro} \right)$$

Bradshaw, P., 1969. The analogy between streamline curvature and buoyancy in turbulent shear flow. J. Fluid Mech. 36, 177–191. <https://doi.org/10.1017/S0022112069001583>

Mininni, P., A. Alexakis, and A. Pouquet: Scale interactions and scaling laws in rotating flows at moderate Rossby numbers and large Reynolds numbers, Phys. Fluids 21, 015108 (2009), DOI:10.1063/1.3064122

Esau, I., Davy, R., Outten, S., Tyuryakov, S., Zilitinkevich, S., 2013. Structuring of turbulence and its impact on basic features of Ekman boundary layers. Nonlinear Process. Geophys. 20, 589–604. <https://doi.org/10.5194/npg-20-589-2013>



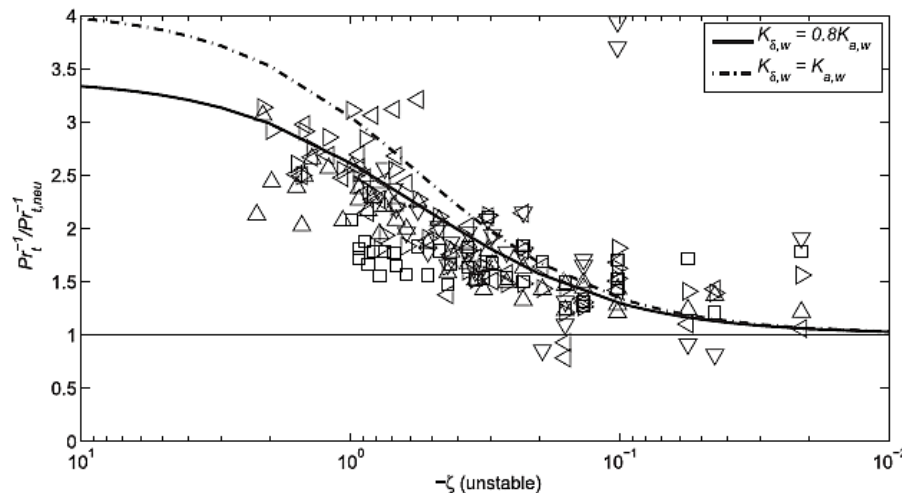
Non-locality: Waves

Physical mechanisms responsible for non-local features of the long-lived SBL turbulence are identified as:

- ***radiation of internal waves from the SBL upper boundary to the free atmosphere, and***
- ***the internal-wave transport of the squared fluctuations of velocity and potential temperature***

The third-order wave-induced fluxes ... correct the wind and temperature profiles in the surface layer. The model explains why developed turbulence in the surface layer can exist at much larger Richardson numbers than the classical theory predicts.

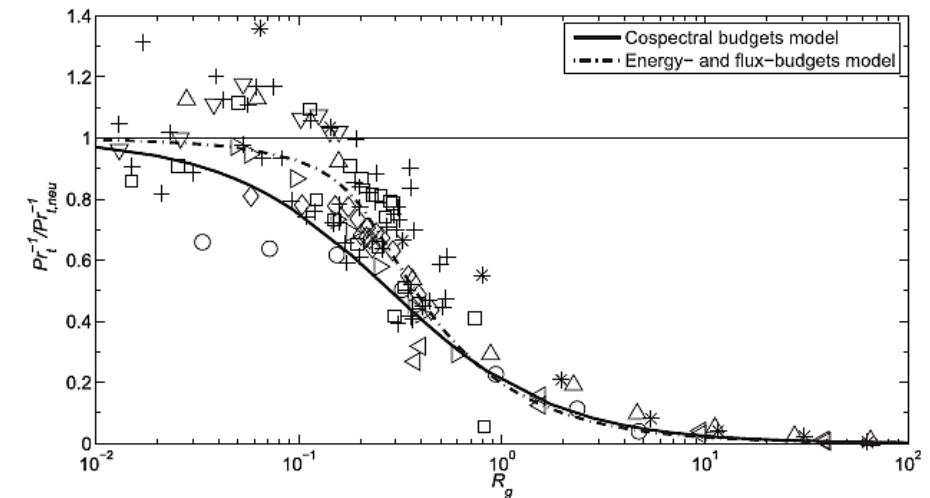
Zilitinkevich, S.S., 2002. Third-order transport due to internal waves and non-local turbulence in the stably stratified surface layer. Q. J. R. Meteorol. Soc. 128, 913–925. <https://doi.org/10.1256/0035900021643746>



Accounting for internal waves naturally leads to ideas of:

- Energy conversion between turbulent kinetic, TKE, and potential, TPE, energy components
- Dependence on imposed stratification
- Differences in transport of TKE and TPE (squared temperature fluctuations)
- Changing turbulent Prandtl number

Li, D., Katul, G.G., Zilitinkevich, S.S., 2015. Revisiting the Turbulent Prandtl Number in an Idealized Atmospheric Surface Layer. J. Atmos. Sci. 72, 2394–2410. <https://doi.org/10.1175/JAS-D-14-0335.1>



Total Turbulent Energy: A paradigm shift

Energy- & flux-budget (E-F-B) theory (2007-2012)

Budget equations for major statistical moments

Turbulent kinetic energy (TKE)	E_K
Turbulent potential energy (TPE)	E_P
Vertical flux of temperature	$F_z = \langle \theta w \rangle$ [or buoyancy $(g/T)F_z$]
Vertical flux of momentum	$\tau_{iz} = \langle u_i w \rangle$ ($i = 1, 2$)
	$t_T = l_\varepsilon(\text{TKE})$ $l_\varepsilon \neq l$

New relaxation equation for the dissipation time scale

Accounting for TPE vertical flux of buoyancy (that killed TKE in Kolmogorov's type closures) **drops out from the equation for total turbulent energy ($TTE = TKE + TPE$)**

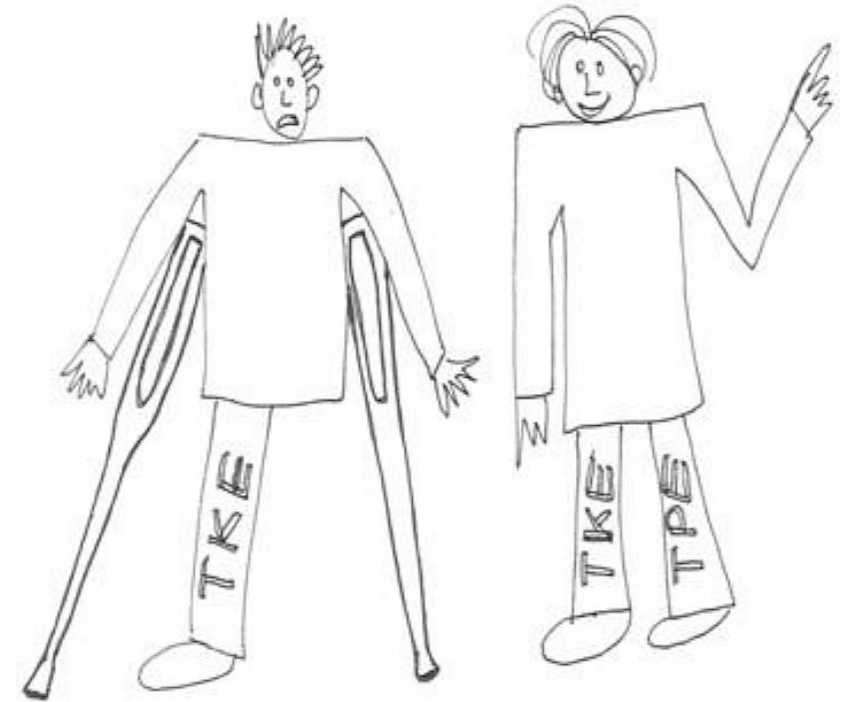
The heat-flux equation reveals a self-limitation of the vertical heat/buoyancy flux causing essential **self-preservation of turbulence up to** $Ri \sim 10^2$

Physical mechanisms and concepts

- Kolmogorov's model for the effective dissipation of the turbulent flux of momentum
- Non-gradient generation of the buoyancy flux \rightarrow **self-preservation of turbulence**
- New, physically consistent model of the turbulent dissipation time / length scales
- Fully revised inter-component energy exchange (instead of "return to isotropy")

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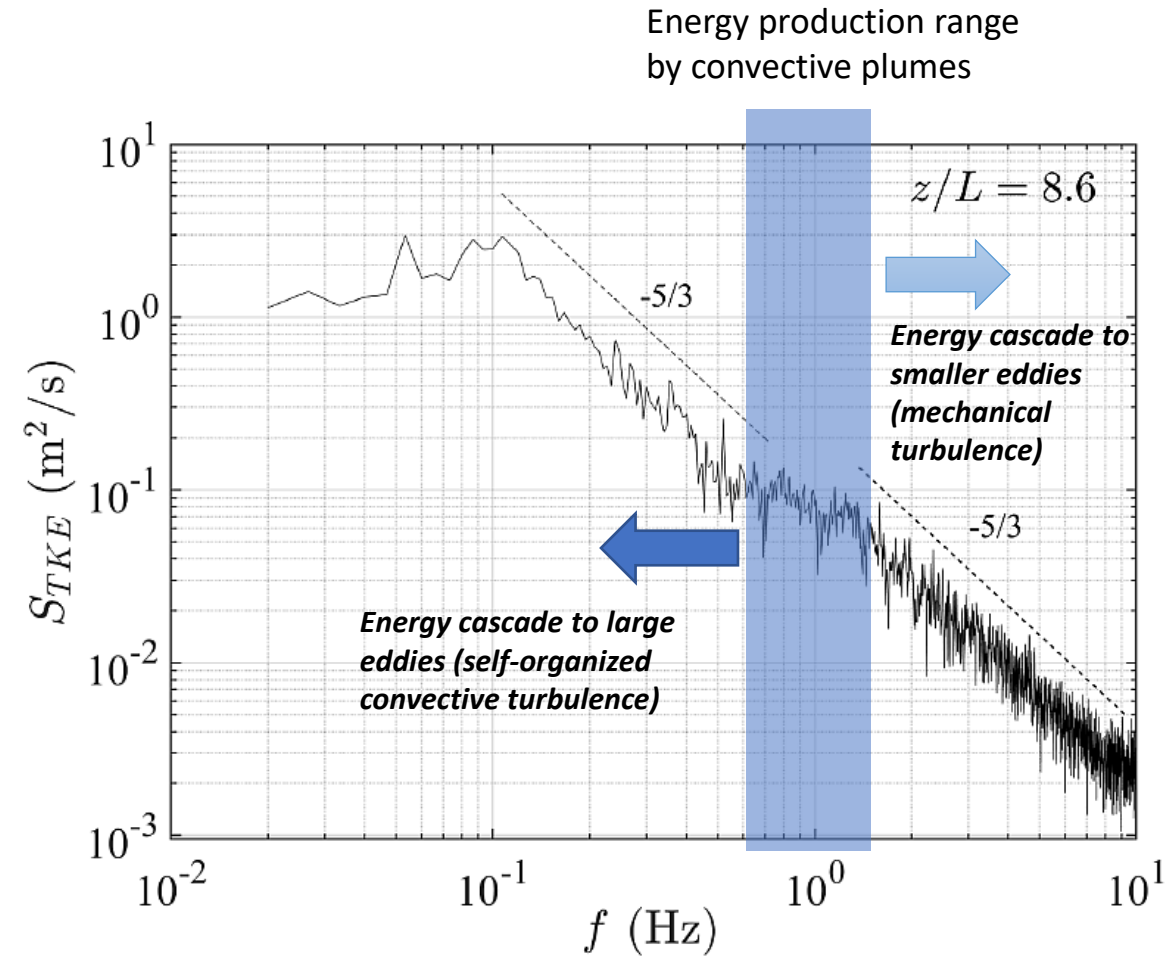
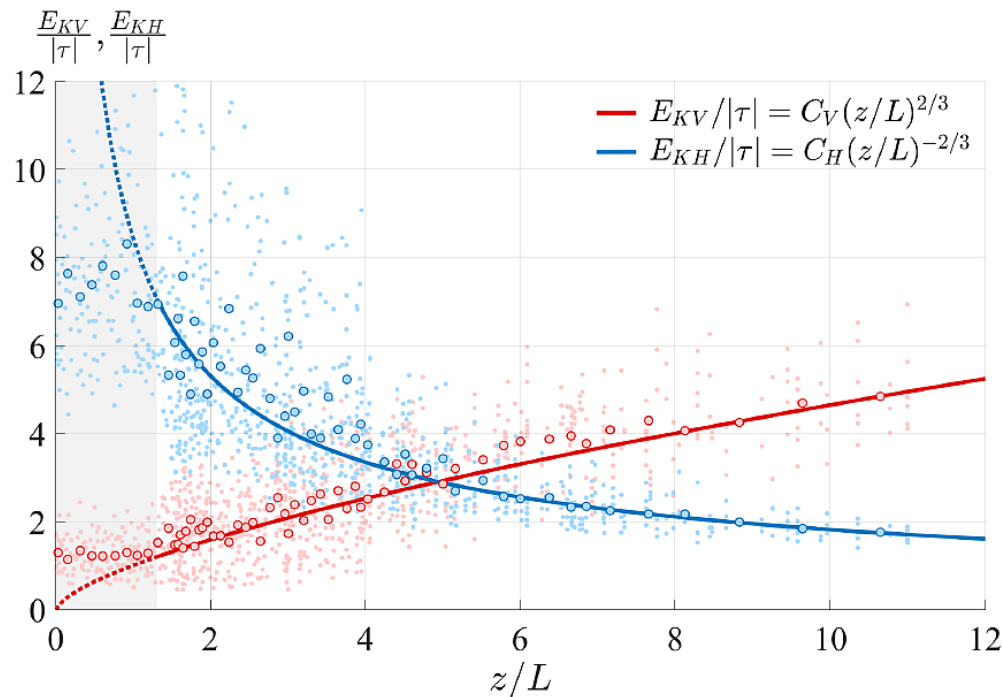


The earliest note on TKE and TPE in the turbulent energy context:
Pristley, C.H.B., Swinbank, W.C., 1947. Vertical transport of heat by turbulence in the atmosphere. Proc. R. Soc. London. Ser. A. Math. Phys. Sci. 189, 543–561. <https://doi.org/10.1098/rspa.1947.0057>

Non-locality: Eddies

The proposed ***new paradigm*** declares the principal difference between the two types of chaotic motions:

- convective turbulence consisted of vertical buoyant plumes, which merge to produce larger plumes, thus producing inverse cascade
- mechanical turbulence consisted of the shear-generated eddies, which move in all directions and break down to produce smaller eddies, thus making direct cascade



Comparison of dimensionless vertical profiles of the fully mechanical horizontal TKE, E_{KH} , and the dominantly convective vertical TKE, E_{KV} .

Sergej Zilitinkevich

13.04.1936 – 15.02.2021

<https://peexhq.home.blog/2021/03/26/to-the-memory-of-sergej-zilitinkevich/>

