Time-dependent solutions for daily-periodic slope flows driven by surface energy budget

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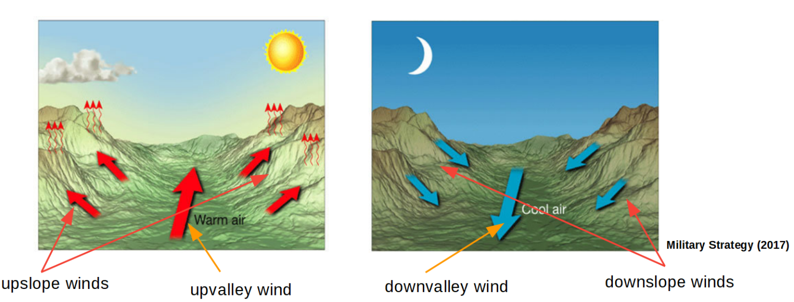
Introduction

Thermally-driven winds

→ Originate from the daytime heating/nighttime cooling of sloping surfaces (more details on transition in Farina et al. 2021, EMS2021)

→ Mostly occur during clear-sky summer days characterized by weak synoptic forcing

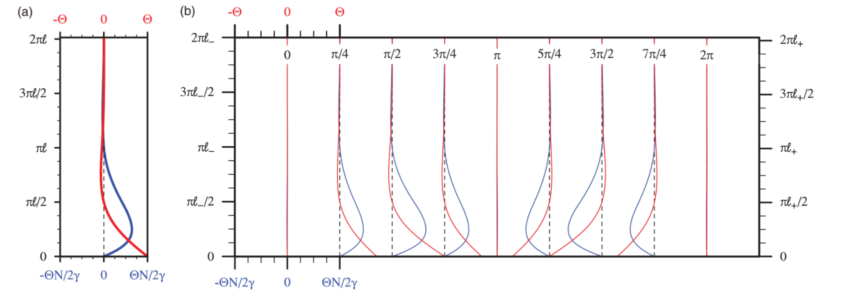


Existing analytical models for slope winds

Most known analytical models describing the profile of slope winds are of two types :

→ Stationary in time (e.g. Prandtl 1942)

→ Time-dependent: Zardi and Serafin (2015). Temperature anomaly and along slope wind profile as a response to a Sinusoidal temperature forcing at the surface.

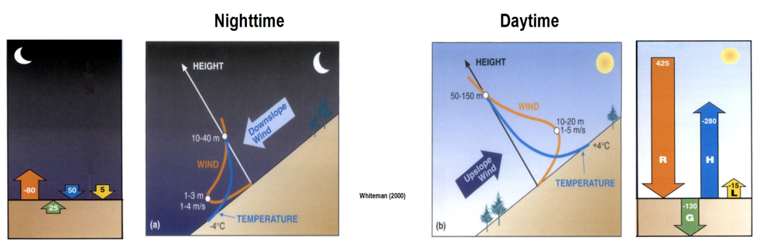


Zardi and Serafin (2015), temperature anomaly (red) and along-slope wind (blue) for large slope angles.

Problems in existing time-dependent model

→ Sinusoidal temperature at the surface produces a symmetric up-slope down-slope regime

→ Observed profiles of slope winds are asymmetric between daytime and nighttime regimes.



→ The Surface energy budget governs the onset and structure of slope winds.

Can we use it as boundary condition for surface temperature?

Zardi and Serafin (2015) model

Governing equations:

$$\frac{\partial \overline{u}}{\partial t} = \overline{\theta}\frac{N^2}{\gamma}sin(\alpha) + K_m\frac{\partial^2 \overline{u}}{\partial n^2} \qquad \frac{\partial \overline{\theta}}{\partial t} = -\overline{u}\gamma sin(\alpha) + K_h\frac{\partial^2 \overline{\theta}}{\partial n^2} \tag{1}$$

Surface boundary conditions:

$$\overline{u}(0,t) = 0 \qquad \overline{\theta}(0,t) = \Theta sin(\omega t + \psi) \tag{2}$$

Given

$$\omega_+ = N_\alpha + \omega \qquad \omega_- = N_\alpha - \omega \qquad l_+ = \left(\frac{2K}{\omega_+}\right)^{1/2} \qquad l_- = \left(\frac{2K}{\omega_-}\right)^{1/2} \tag{3}$$

Solutions for the supercritical case ($N_\alpha > \omega$, strong stability and steep slopes) hold:

$$\overline{u} = \frac{\Theta}{2}\frac{N}{\gamma}\left[e^{-n/l_+}cos\left(\omega t - \frac{n}{l_+} + \psi\right) - e^{-n/l_-}cos\left(\omega t + \frac{n}{l_-} + \psi\right)\right] \tag{4}$$

$$\overline{\theta} = \frac{\Theta}{2}\left[e^{-n/l_+}sin\left(\omega t - \frac{n}{l_+} + \psi\right) - e^{-n/l_-}sin\left(\omega t + \frac{n}{l_-} + \psi\right)\right] \tag{5}$$

Extension to a more general periodic surface forcing

The surface temperature boundary condition was modified as:

$$\overline{\theta}(0,t) = \sum_{m=1}^{\infty} \Theta_m sin(m\omega t + \psi_m) \tag{6}$$

and solutions for temperature and wind velocity:

$$\begin{aligned}\overline{u} =& \sum_{m=1}^{c} \frac{\Theta_m}{2}\frac{N}{\gamma}\Big[e^{-n/l_{m+}} cos\left(m\omega\hat{t} - \frac{n}{l_{m+}} + \psi\right) - e^{-n/l_{m-}} cos\left(m\omega\hat{t} + \frac{n}{l_{m-}} + \psi\right)\Big] \\ &+ \sum_{m=c+1}^{\infty} \frac{\Theta_m}{2}\frac{N}{\gamma}\Big[e^{-n/l_{m+}} cos\left(m\omega\hat{t} - \frac{n}{l_{m+}} + \psi\right) - e^{-n/|l_{m-}|} cos\left(m\omega\hat{t} - \frac{n}{|l_{m-}|} + \psi\right)\Big]\end{aligned} \tag{7}$$

$$\begin{aligned}\overline{\theta} =& \sum_{m=1}^{c} \frac{\Theta_m}{2}\Big[e^{-n/l_{m+}} sin\left(m\omega\hat{t} - \frac{n}{l_{m+}} + \psi\right) - e^{-n/l_{m-}} sin\left(m\omega\hat{t} + \frac{n}{l_{m-}} + \psi\right)\Big] \\ &+ \sum_{m=c+1}^{\infty} \frac{\Theta_m}{2}\Big[e^{-n/l_{m+}} sin\left(m\omega\hat{t} - \frac{n}{l_{m+}} + \psi\right) - e^{-n/|l_{m-}|} sin\left(m\omega\hat{t} - \frac{n}{|l_{m-}|} + \psi\right)\Big]\end{aligned} \tag{8}$$

Harmonics higher than some critical value (ω_c) are subcritical.

Solution of the surface energy budget

Surface energy budget holds:

$$R_{net} = (1 - A)R_{in} + LW_{net} = H + G = -K \left.\frac{\partial\overline{\theta}}{\partial n}\right|_{n=0} - K_G \left.\frac{\partial\overline{T}}{\partial n}\right|_{n=0} \tag{9}$$

Being the temperature signal in the ground:

$$\overline{T} = \sum_{m=1}^{\infty} \Theta_m sin\left(m\omega\hat{t} + \frac{n}{l_{Gm}} + \psi\right) e^{n/l_{Gm}} \tag{10}$$

Equation (9) becomes:

$$R_{net} = \sum_{m=1}^{\infty} \Theta_m \sqrt{a_m^2 + b_m^2} sin(m\omega\hat{t} + \sigma_m + \phi_m) \tag{11}$$

$$a_m = \frac{K}{2l_{m+}} - \frac{K}{2l_{m-}} - \frac{K_G}{2l_{Gm}} \qquad b_m = \frac{K}{2l_{m+}} + \frac{K}{2l_{m-}} - \frac{K_G}{2l_{Gm}}$$
$$\phi_m = arcsin\left(\frac{b_m}{\sqrt{a_m^2 + b_m^2}}\right) \tag{12}$$

$\rightarrow$ **If R_{net} can be written as a sum of sines too, Θ_m can be derived for each harmonic to give the surface temperature signal controlled by the energy balance.**

Net Radiation model

Net Radiation Budget

Net radiation budget can be written as

$$R_{net} = R_{in} \ + \ R_{out} \ + \ LW_{in} \ + \ LW_{out} \tag{13}$$

Assumptions:

- R_{in} considers just the direct component
- R_{out} is computed by considering a mean albedo of the surface
- $LW_{net} = \epsilon T_a^4 - \epsilon T_s^4$ will be treated as a constant

Incoming solar radiation at the surface:

$$R_{in} = R_s \ \cos\theta \ \tau \tag{14}$$

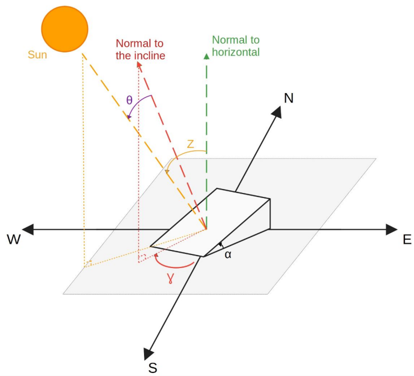
where R_S is the solar radiation at the top of the atmosphere, $\cos\theta$ is the cosine solar incident angle (θ) and τ is the atmospheric transmittance.

Incoming solar radiation in complex terrain

- Incoming radiation mainly controlled by latitude and day of the year.
- Topography (slope angle and orientation) also plays an important role

$$\cos\theta = (\sin\phi\ \cos\alpha - \cos\phi\ \sin\alpha\ \cos\gamma)\ \sin\delta + (\cos\phi\ \cos\alpha + \sin\phi\ \sin\alpha\ \cos\gamma)\ \cos\delta\ \cos(\omega\hat{t}) + \cos\delta\ \sin\gamma\ \sin\alpha\ \sin(\omega\hat{t}) \tag{15}$$

$$\hat{t} = t - 12h \qquad \text{Time with respect to solar noon } (\hat{t} = 0) \tag{16}$$



Clear-sky atmospheric transmittance

Atmospheric transmittance is parameterized using the model of Hottel (1976):

$$\tau = \tau_0 + \tau_1 \cdot exp\left(-\frac{k}{cos(Z)}\right) \tag{17}$$

$$\cos(Z) = \sin\phi\ \sin\delta\ +\ \cos\phi\ \cos\delta\ cos(\omega\hat{t}) \tag{18}$$

$\rightarrow$ for writing the radiation as a series expansion model, we tested two approximations of the exponential term

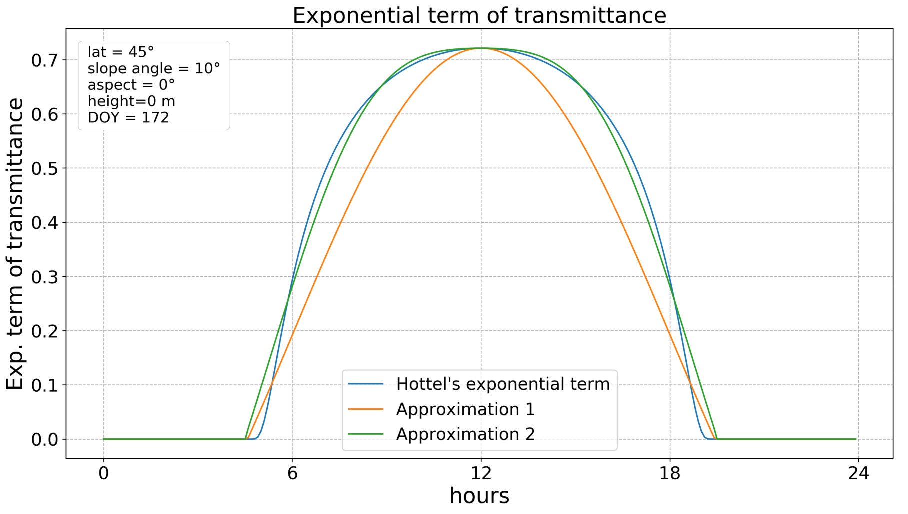
- Approximation 1:

$$\begin{aligned} exp\left(-\frac{k}{\cos Z}\right) &= \frac{1}{F_c}\cos Z\ exp(-k) \\ F_c &= exp\left(k\left(\frac{1}{\cos(\phi-\delta)}\right)-1\right)\cdot\cos(\phi-\delta) \end{aligned} \tag{19}$$

- Approximation 2:

$$\begin{aligned} exp\Big(-\frac{k}{\cos Z}\Big) &= \frac{1}{F_c}\Big(c_1\cos Z\ c_2\cos 2Z\ +\ c_3\cos 3Z\Big)exp(-k) \\ F_c =& exp\Big(k\Big(\frac{1}{\cos(\phi-\delta)}\Big)-1\Big)\cdot\Big(c_1\cos(\phi-\delta)\ +c_2\cos(2(\phi-\delta)) \\ &+c_3\cos(3(\phi-\delta))\Big) \qquad c_1=5,\ c_2=-0.1,\ c_2 = \text{-}0.5 \end{aligned} \tag{20}$$

Clear-sky atmospheric transmittance

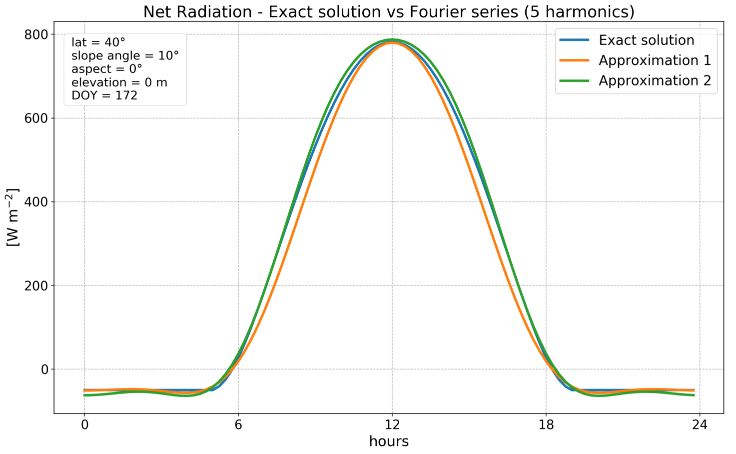


Net Radiation outcome with the two approximations

The net radiation budget can be written as:

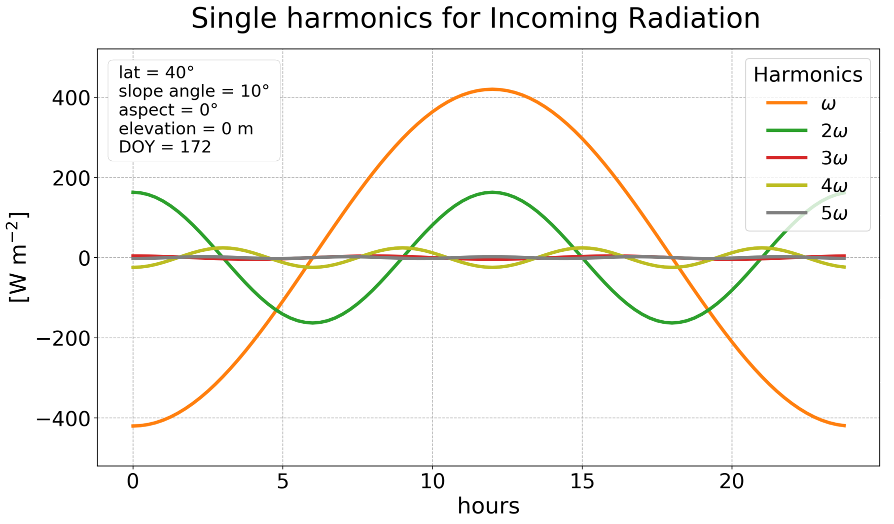
$$R_{net} = (R_0) + \sum_{m=1}^{\infty} \sqrt{R_{mc}^2 + R_{ms}^2}\,\mathbf{sin}(m\omega\hat{t} + \sigma_m + \psi_m) \tag{21}$$

$\rightarrow$ Equalling Eq. (19) to Eq. (11) gives the SFC temperature. R_0 includes LW and originates a transient signal.



Relative importance of harmonics

→ 2 harmonics are enough to reconstruct the radiation profile



Wind and temperature profiles

Case study

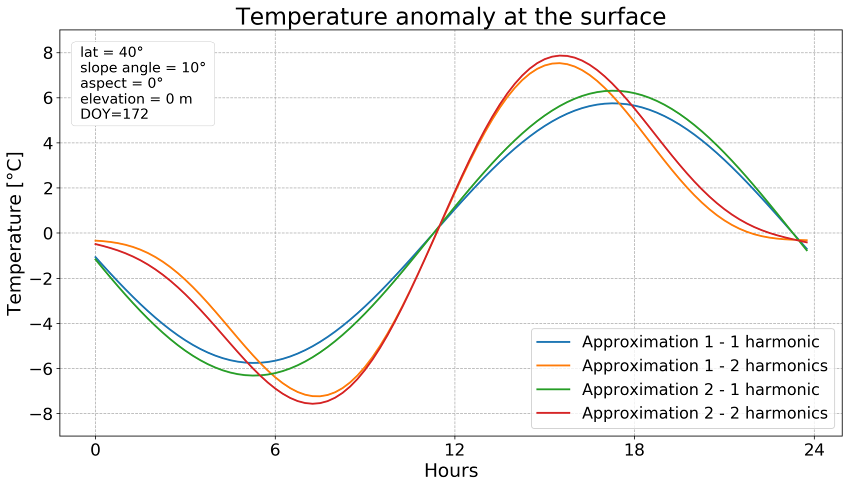
Parameters used are:

- Slope angle = 10 °
- Slope orientation = 0° (south facing slope)
- Day Of the Year (DOY) = 72 (Summer solstice)
- Eddy diffusivity (kinematic) = 3 m^2s^{-1}
- Heat diffusivity in soil (kinematic) = 10^{-06} m^2s^{-1}
- atmospheric lapse rate = 0.002 Km^{-1} $\omega = 7.2e-05$

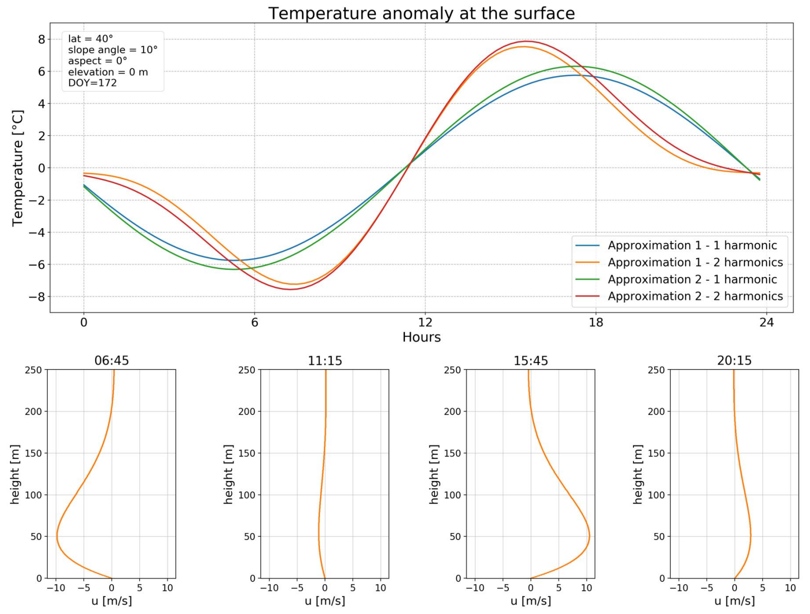
$$N \sin\alpha = 1.4e-03 > 7.2e-05 = \omega$$

$\rightarrow$ Supercritical regime also for the considered higher order harmonics

Surface temperature profile



Surface temperature profile



Conclusions

Conclusions and perspective works

- The improved surface boundary condition for temperature accounts for the usually observed difference in magnitude between the daytime and nighttime regimes of slope winds
- In real situation the depth of the slope wind layer is governed by eddy diffusivity, which assumes different values between night and day.
- The work shows the effect of zero-mean, daily periodic temperature anomaly. Transient temperature signal from constant flux term (from Fourier expansion series) should be taken into account too.
- Further developments including non-constant eddy viscosity and more complete surface forcing are expected to provide more realistic solutions.

Thank you for your attention!

Feel free to contact me at mattia.marchio@unitn.it for any question or curiosity.

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