Extremal dependence as given by the tail pairwise dependence matrix in precipitation and temperature data

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CoDEX — Compact Description and Statistical Modelling of Extremes — Universities of Bonn and Stuttgart

Compact statistical-dynamical description of high-dimensional spatial weather extremes

The CoDEx project is investigating how data compression techniques can contribute to a better description and understanding of extreme weather events.
We use singular value decomposition to study the spatial co-occurrence of temperature and precipitation during heat wave-events.
We identify a spatial relationship between both variables and obtain a temporal identification of prominent heat waves in Europe. We also achieve improved compact representation of the temperature time series for extremes.
The focus on extreme events is done by using the tail pairwise dependence matrix (TPDM), proposed by Cooley and Thibaud (2019) as an analogue to the covariance matrix for extremes. To capture the simultaneous occurrence of precipitation deficits and high temperature, we explore how Cooley and Thibaud's concept can be used in this regard and propose an estimation of the TPDM based on pairwise dependencies of two variables.

Motivation

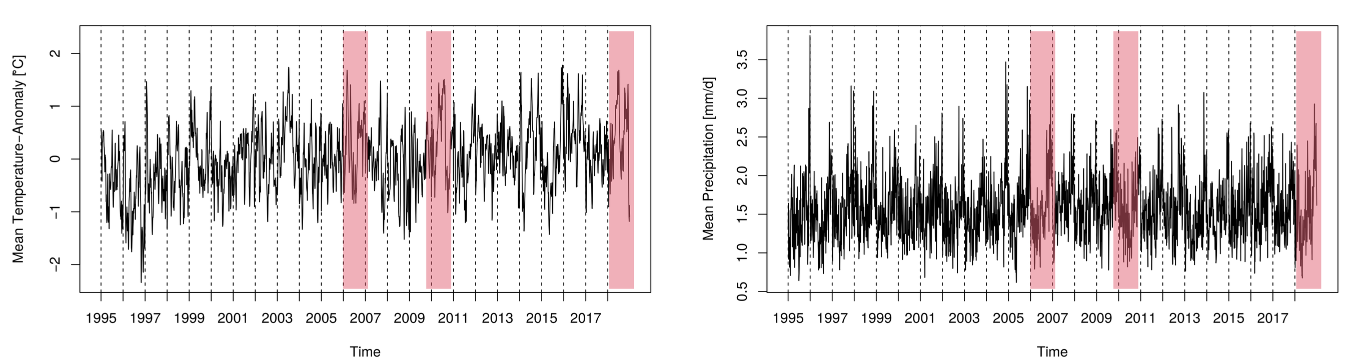

Figure 1. Spatial mean of temperature-anomalies (left) and precipitation (right) over CORDEX-11 Domain. The Years 2003, 2010 and 2018 are highlighted in red.

- **High-dimensionality** of meteorological datasets (spatial/temporal/multivariate)
- low **signal-to-noise ratio** (curse of dimensionality)

Adapt information compression/ dimension reduction to meteorological extremes!

A Singular Value Decomposition using Extremal Dependencies

Various data compression techniques are based on the covariance-matrix. To achieve a targeted application to extremes, we follow [2] and **replace the covariance-matrix by the TPDM** (Tail Pairwise Dependence Matrix) which is refered to as an analogue to the covariance matrix, but for extremal dependence.

- The estimate of the TPDM is a **threshold based estimation**
- The definition of the TPDM is based on an **Extreme framework**

Dataset: COSMO-REA6

- Domain: CORDEX EURO-11
- Resolution: 0.055° (6 km)
- Period: 1995-2019
- `http://reanalysis.meteo.uni-bonn.de`
- Daily **Preciptiation**, **Temperature**
- **Extended Summer:** June-September
- 1995-2019
- Detrended Temperature

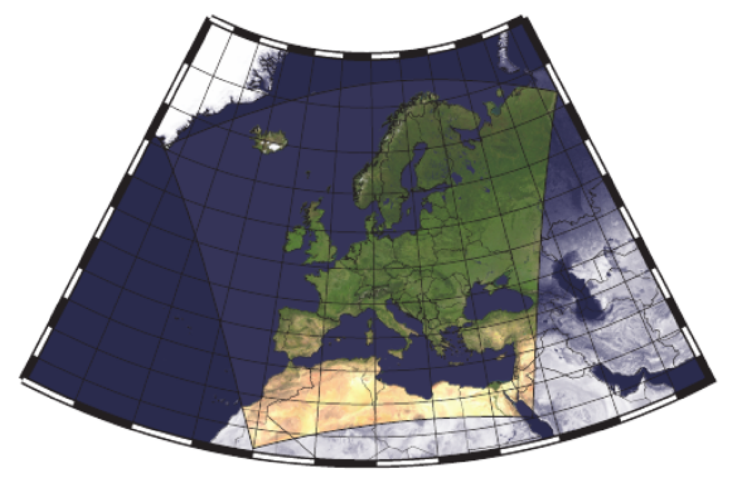

Figure 2. Representation of the CORDEX EURO-11 domain [Figure from 1]

Definition of the TPDM

Assume **regular varying** random vectors $X \in RV_+^p(2)$ (and $Y \in RV_+^q(2)$) such that:

$$nPr(n^{-1/2}X \in \cdot) \to^v \nu_X(\cdot) \quad \nu_X(dr \times dw) = 2r^{-3} dr dH_X(w)$$

Define each element of the TPMD after a **polar transform**:

$$\boxed{\sigma_{X_{ik}} = \int_{\theta_{p-1}^+} w_i w_k dH_X(w)} \quad \Sigma_X = (\sigma_{X_{ik}})_{i,k=1,\dots,p}$$

With H_X: Radon measure on L_2 unit ball $\theta_{p-1}^+ = w \in \bar{\mathbb{X}}^p : ||w||_2 = 1$

Workflow

1. Transform Original Data such that $X \in RV_+^p(2)$ and $Y \in RV_+^q(2)$
2. Estimate of the TPDM:
$$\hat{\sigma}_{i,j} = 2n_{ij,exc}^{-1} \sum_{t=1}^{n_{samp}} w_{t,i}^X w_{t,j}^Y \mathbb{I}(r_{t,ij} > r_{0,ij})$$
With: $\omega_{t,i}^X = \frac{x_{t,i}}{r_{t,ij}}$; $\omega_{t,j}^X = \frac{x_{t,j}}{r_{t,ji}}$; $r_{t,ij} = \sqrt{x_{t,i}^2 + y_{t,j}^2}$.
3. Compute Singular Value Decomposition (SVD): $\hat{\Sigma}_{XY} = U \Lambda V^T$
4. Analyse:
   - Singular Vectors: $e_i = t(U_i)$; $f_i = t(V_i)$
   - Singular Values: λ_i
   - Expansion Coefficients: $\mu_t = U^T t^{-1}(x_t)$; $\nu_t = V^T t^{-1}(y_t)$
   - Reconstruction: $x_t^\star = U^\star \cdot \mu_t^\star$; $y_t^\star = V^\star \cdot \nu_t^\star$

   with $t(z) = log(1 + \exp(z))$

Results

Expansion Coefficients

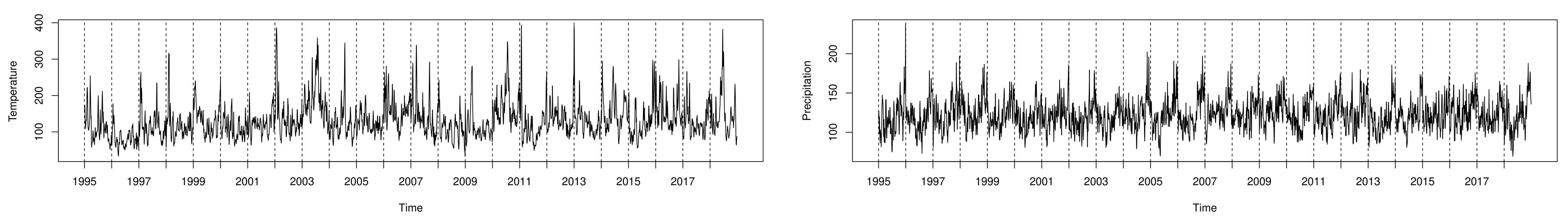

Figure 3. First left- and right expansion coefficients for temperature (left) and precipitation (right). The timeline corresponds to the extended summer months (June-September) and the vertical lines denote the cut between two different years.

- First mode: Temporal identification of Heat Wave Events
- Higher order: Information of Spatial Extend of Extreme Event
- Identification of Extreme Precipitation difficult
- Comparable smaller scales of extreme precipitation events

Singular Vectors

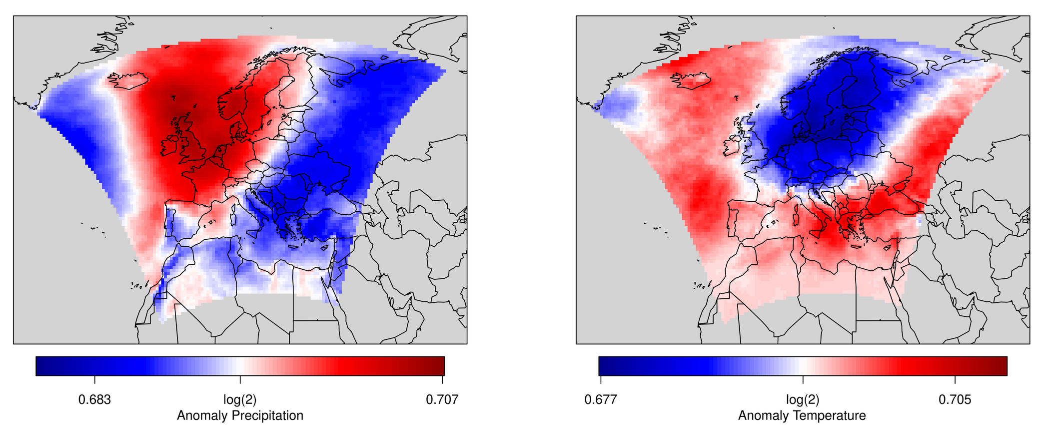

Figure 4. Second singular vectors obtained by the SVD. The anomalies are associated with temperature-anomalies (left) and precipitation-anomalies (right). The representation takes place in the positive reals. Therefor $log(2)$ corresponds to zero.

- Detection of spatial patterns with commonly occurring extremes
- First Singular Vectors entirely positive
- Wave-Like Structure of singular Vectors
  - Relation to Rossby waves
  - Identification of frontal structure

→ Dynamical Link between precipitation and temperature

Summary/ Discussion

- Providing a targeted application of the SVD to extremes
- Allowing to benefit from the experience and knowledge of the method
- Improved compact description for extreme events compared to usual SVD

References

[1] C Bollmeyer, JD Keller, C Ohlwein, S Wahl, S Crewell, P Friederichs, A Hense, J Keune, S Kneifel, I Pscheidt, et al. Towards a high-resolution regional reanalysis for the european cordex domain. *Quarterly Journal of the Royal Meteorological Society*, 141(686):1–15, 2015.

[2] D Cooley and E Thibaud. Decompositions of dependence for high-dimensional extremes. *Biometrika*, 106(3):587–604, September 2019.

[3] Yujing Jiang, Daniel Cooley, and Michael F Wehner. Principal Component Analysis for Extremes and Application to U.S. Precipitation. *JOURNAL OF CLIMATE*, 33:11, 2020.