

# Libration-driven flows in planetary bodies

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## Abstract

Astrophysical bodies are subject to various harmonic forcings such as latitudinal or longitudinal librations, precession or tides. In this study we focus on the longitudinal libration forcing which takes place in planetary bodies such as Ganymede or Europa. Libration measurements can be used to constrain interior models by comparison with theoretical models. The accuracy of these constraints directly depends on the refinement of the model. Taking into account the influence of a non-trivial flow in the internal liquid layers of planetary bodies requires a precise knowledge of the fluid dynamics. In this work we consider a simple model of a librating shell and we focus on: (1) the mean zonal flow induced by a longitudinal libration which is determined using a weakly non-linear analysis; (2) the torque and the viscous dissipation in the interior fluid induced by longitudinal libration, which are numerically studied in various cases where inertial waves may be present or in the presence of a centrifugal instability at the outer boundary.

## 1. Introduction

The librational forcing can lead to important flow in the liquid core and subsurface ocean of telluric planets or in the atmosphere of gaseous planets. In this study, we focus on the longitudinal libration which corresponds to an harmonic oscillation of the rotation rate of a container. These oscillations originate for instance from gravitational coupling between an astrophysical body and its main gravitational partner around which it orbits [1]. The librational forcing can have a non negligible contribution in the dynamic of the internal fluid of a planet and a better knowledge of these dynamics can bring important informations on the structure of a planet. For example, this can provide an estimation of the thickness of the fluid layer under the ice shell in some satellites like Europa, Ganymede or Enceladus [2] or allows to investigate the presence of a liquid core in telluric planets like Mercury [3]. However, the current models do not include the whole fluid dynamics

of the internal liquid layers due to a lack of knowledge of the flow induced by this forcing. The aim of this work is to describe this flow, providing the first step toward a better understanding of a librating planetary body.

## 2. Mean zonal flow in a spherical shell

We model a planetary body by a spherical shell of external radius  $R_{ext}$  (chosen as the length scale), and inner radius  $R_{int}$ , filled with a homogeneous and incompressible fluid of kinematic viscosity  $\nu$  and density  $\rho$ . The mean rotation rate  $\Omega_0$  is used as a time scale. The instantaneous angular velocity of the container is the sum of a constant rotation and a perturbation term;  $\mathbf{\Omega}(t) = [1 + \epsilon \cos(\omega t)] \mathbf{e}_z$ , where  $\epsilon = \omega \Delta\phi$  and  $\omega$  are respectively the dimensionless amplitude and frequency of the librational forcing. The system is determined by four dimensionless numbers: the Ekman number  $E = \nu / (\Omega_0 R_{ext}^2)$  which represents the ratio of viscous force to Coriolis force, the libration frequency  $\omega$ ,  $\epsilon = \omega \Delta\phi$  and the aspect ratio of the shell,  $\alpha = R_{int} / R_{ext}$ .

In astrophysical applications, the Ekman number of planetary bodies and the libration frequency  $\omega$  satisfy the relation  $\omega \gg \sqrt{E}$  for the most important components of the libration spectrum [2]. It means that the interior fluid of the shell does not spin-up and spin-down at each libration cycle. To study the dynamics of the fluid in the interior we first consider the mean zonal flow, which is steady, axisymmetric and azimuthal [4], [5]. Using a weakly non-linear analysis, we show that the mechanism of zonal flow generation is fully generic: the main contribution in the bulk always comes from the non-linear self-interaction of the viscous boundary layer flow induced by the librational forcing. Moreover, in a shell, the presence of an inner core modifies the zonal flow and generates an axial velocity in the cylinder tangent to the inner core through Stewartson layers.

### 3. Torque and viscous dissipation

The complete non-linear flow is then studied using an axisymmetric finite elements code (COMSOL Multiphysics, a commercial software) to solve the non-linear Navier-Stokes equations with no-slip boundary conditions. This code has been successfully used in a previous study [5]. Depending on the libration frequency, the structure of the interior flow can be highly modified by the presence of inertial waves (figure 1), and for amplitude of libration sufficiently large, a centrifugal instability takes place under the form of Taylor-Görtler vortices near the equator of the outer boundary (figure 2). The torque and viscous dissipation related to libration driven flows have been studied systematically to evaluate the influence

(1) of Taylor-Görtler vortices near the sidewall boundary which are expected to be present in certain bodies such as Ganymede or Europa [6].

(2) of inertial waves which may contribute significantly to the interior flow. They are also expected to be present in librating bodies because many libration frequencies are such that  $\omega \in [0; 2]$  [2].

We have also extended this study to the more relevant case of a triaxial ellipsoid, closer to the real shape of planetary bodies.

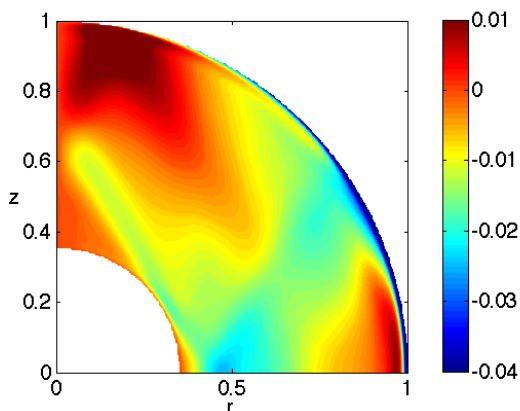


Figure 1: Azimuthal velocity in a librating shell for  $\epsilon = 0.1$ ,  $\omega = 1$ ,  $E = 5.10^{-5}$ . The inertial waves excited by the librational forcing are clearly visible.

### 4. Conclusions

In this work we have studied the dynamics of a fluid in a librating shell. The mean zonal flow driven by libration has been characterized analytically for libration

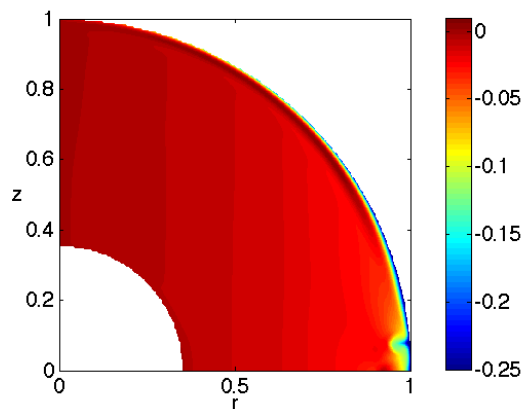


Figure 2: Azimuthal velocity in a librating shell for  $\epsilon = 0.1$ ,  $\omega = 1$ ,  $E = 5.10^{-5}$ . Taylor-Görtler vortices are localized near the equator of the outer sphere.

frequency where inertial waves are absent or negligible. Then astrophysical relevant quantities (the torque at the outer boundary and the viscous dissipation) have been numerically studied in order to obtain relevant scaling laws. The effects of the interior flow are expected to be non-negligible, for example in both the subsurface ocean and the core of Europa,

### References

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