Topography and geoid induced by a convecting mantle beneath an elastic lithosphere

O. Golle (1), C. Dumoulin (1), G. Choblet (1), O. Čadek (2)
(1) University of Nantes, CNRS, Laboratoire de Planétologie et Géodynamique de Nantes, France; (2) Charles University, Department of Geophysics, Prague, Czech Republic; (olivia.golle@univ-nantes.fr / +33-251-125474)

Abstract

Thermal convection that occurs in terrestrial planetary bodies induces density anomalies but also dynamic topographies of the main interfaces. Both contribute to the shape of the geoid. While a classical approach now is to combine gravity and altimetry measurements to infer the internal structure of a planet [1], our complementary approach consists in computing synthetic dynamic topography and geoid from thermal convection calculations in order to understand their relationship. Here, we couple the deformation of an elastic shell (mimicking a planetary lithosphere) with the viscous convective flow below it. The viscous flow is computed using a 3D numerical tool for a spherical shell (OEDIPUS [2]) using a finite difference method that allows large lateral viscosity variations. The deformation of the elastic layer is computed using a semi-spectral method.

We show that introducing the total traction force (instead of a simplified coupling involving only the radial component of the traction force as often assumed in earlier studies) results in a larger filtering effect caused by the elastic lithosphere (especially for thin elastic layers). In a last step, we will apply our hybrid tool to simple thermal convection calculations and compute the associated topography and geoid maps. Spectral characteristics of these synthetic signals are presented and discussed.

1. Model description

The model presented here focuses on the effect of a cold and viscous lithosphere that behaves as an elastic medium on geological timescales, on the dynamic topography and geoid resulting from thermal convection within a planetary mantle. The numerical method we propose describes heat transfer in a spherical shell as well as two distinct deformation processes associated to two different sublayers resulting from the partition of this global shell: (i) elastic deformation of a thin superficial shell (from radius $R_m$ to $R_s$) and (ii) viscous flow in the deeper region (from radius $R_s$ to $R_m$).

1.1. Conservation of Energy

We solve the equation of conservation of energy that describes the thermal evolution of a fluid layer of infinite Prandtl number heated volumetrically and/or from below. The Boussinesq approximation is considered and viscous dissipation is neglected. The treatment of the conservation of energy is included in the finite volume numerical tool OEDIPUS [2] solving thermal convection in a spherical shell.

1.2. Flow in the viscous shell

We consider viscous flow in the subdomain bounded by spheres of radii $R_s$ and $R_m$, i.e. beneath the shallow shell that is treated as an elastic medium. The Stokes equations describing conservation of mass and momentum are solved using a multigrid method with the same tool OEDIPUS as for the conservation of energy. Note however that for the specific, hybrid method described in the present study, the location of the upper boundary for the flow solver ($r = R_m$) is below the boundary used for the temperature field ($r = R_s$) if an elastic shell of thickness $d_e = R_s - R_m$ is introduced. Viscosity follows a Arrhenius law with a strong temperature and pressure dependence.

1.3. Deformation of the elastic shell

The elastic shell is located between radii $r = R_m$ and $r = R_s$. The boundary condition at the lower boundary $r = R_m = R_s - d_e$ is formulated in terms of the traction force $F$ resulting from the viscous flow and therefore computed at the surface of the viscous layer. Equations governing small deformations of an incompressible elastic body of constant density are solved together with the boundary conditions in the spectral domain.
Dynamic bottom (induced by the viscous flow) and surface (induced by the viscous flow but filtered by the elastic shell) topographies as well as geoid anomalies are then computed, in the spectral domain.

2. Results

Previous studies [3, 4] using a similar basic configuration as here consider a simplified model of the mechanical coupling between the shells: only the radial component of the traction force that arises from the viscous flow, $F_r$, is used to compute the deformation of the elastic shell while the tangential component, $F_t$, is neglected. We introduce elastic filter coefficients $C_\ell$ and $S_\ell$ describing the relationship between the forces $F_r$ and $F_t$ acting at the base of the shell and the resultant surface topography $t_s$:

$$t_s^m = -C_\ell(F_r)_m - (F_t)_m.$$  

Fig. 1 shows variations of

\begin{align*}
\text{a)} & \quad C_\ell \\
\text{b)} & \quad S_\ell
\end{align*}

Figure 1: Elastic filter coefficients a) $C_\ell$ and b) $S_\ell$ computed for three generic bodies - Venus, Mars and Dione. Two elastic lithosphere thicknesses are considered for each body.

The elastic filter coefficients for three generic bodies degree by degree. It is worth noting that the filtering effect strongly differs for different solar system bodies. At low degrees, the values of $S_\ell$ are smaller than those of $C_\ell$ but their relative importance may increase with increasing degree. Although the value of $S_\ell$ is always positive, the tangential force tends to diminish the surface topography because the ratio $\langle F_t \rangle_m / \langle F_r \rangle_m$ is negative in most cases. The contribution of the tangential traction force to the filtering effect might therefore not be negligible, especially for thin elastic shells (see for example Fig. 2).

\begin{align*}
\text{a)} & \quad \text{Surface topography computed a) using the radial approximation and b) using both radial and tangential components of traction force arising from the viscous flow, for a Mars body-like and an elastic shell of 100 km.}
\end{align*}

References


