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Three-dimensional asymptotic thermal model of a rotating comet nucleus

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Abstract

The method of matched asymptotic expansions is applied for analysis of a generic unsteady three-dimensional (3D) thermal model of a rotating and sublimating spherical comet nucleus. The properly generalized Van Dyke's matching rule, accounting for non-linearity of the boundary condition, results in an asymptotic model, which provides accurate description of the temperature field even if the Fourier number Fo based on the nucleus period of revolution is as small as 10. In a large range of conditions, the surface temperature field and the gas production rate are independent of Fo, but determined by a new "rotational" parameter, which serves as measure of the nucleus rotation rate. The accuracy of the asymptotic models is checked by the comparison of the computational results obtained based on the full 3D and asymptotic models. It is found that the asymptotic model is capable for computationally-efficient prediction of the temperature field at arbitrary depth inside a nucleus in a whole range of parameters characteristic for comets in the Solar system.

1. Introduction

The rotation is of importance for the thermal state and outgassing of rotating cometary nuclei [1]. In spite of this fact, most studies of the near-nucleus cometary atmospheres were performed to date, assuming axially-symmetric surface temperature distribution and near-nucleus flow and neglecting the rotation effects, e.g., Refs. [2,3]. Combined simulations of the three-dimensional thermal state of a nucleus, its outgassing, and near-nucleus atmosphere [4] require computationally-efficient and quantitatively-predictive models of the nucleus heat and mass transfer.

If the period of revolution is short as compared with the characteristic time needed for the heat propagation deep inside a nucleus, the diurnal variations of temperature affect only a thin boundary layer at the nucleus surface In comet nuclei, this thermal boundary layer can be as thin as a few centimeters on a few kilometer-size nucleus [5], but the structure of this layer is of primary importance for the prediction of the nucleus gas production rate. Small thickness of the boundary layer makes impractical or even infeasible the application of full 3D models for calculations of the temperature field in rotating nuclei [6,7]. Instead, a variety of approximate quasi-3D models have been proposed based on an intuitive assumption that the effect of thermal conductivity is important only in the direction that is perpendicular to the body surface, e.g., Refs. [5,8-10].

In the present work, in particular, it is shown that this simple assumption is not necessarily correct, and there are thermal regimes, when thermal conduction in the latitudinal direction can not be neglected even if the Fourier number, Fo, based on the body period of revolution is large. On the other hand, it is shown that the accurate asymptotic analysis of the problem results in a computationally-efficient asymptotic model, which accounts for the heat transfer in the latitudinal direction. The proposed model is found to be capable for accurate prediction of the gas production rate and the nucleus thermal structure in a whole range of parameters characteristic for comets in the Solar system.

2. Model and results

The temperature field in a spherical nucleus is described by the unsteady heat conduction equation with a strongly non-linear boundary condition, accounting for the light absorption, thermal conductivity, re-radiation, and sublimation on the sphere surface [7]. The problem is formulated in a dimensionless form, where the solution is a function of the Fourier number and a few other dimensionless parameters. Using the method of matched asymptotic expansions [11], the nucleus temperature field is represented in a form, combining the temperature fields inside the thermal boundary layer and in the inner part of the nucleus. It is shown that there is strong coupling between both parts of the temperature field due to non-linear boundary condition on the nucleus surface [12]. Accounting for this coupling and using a properly generalized Van Dyke's matching rule, an asymptotic model is developed, that is capable for accurate prediction of the temperature field in the whole nucleus even if the Fourier number is as small as 100 (Fig. 1). The developed asymptotic model is also capable for accurate description of the thermal boundary layer degeneration, when the angle between the rotational axis and the direction to the light source tends to zero and the effect of rotation vanishes.

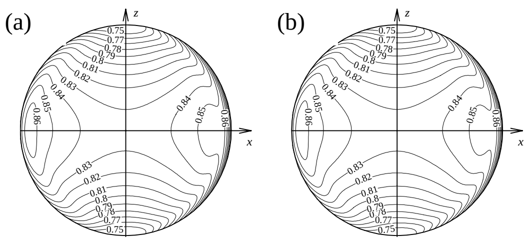

Figure 1: Contours of constant temperature in the nucleus cross section obtained at $Fo = 100$ with the asymptotic model (a) and the full 3D model (b). The nucleus rotates around axis *z*.

4. Summary and conclusions

From the computational point of view, the developed asymptotic model is superior to the full 3D model, since computational efficiency of the asymptotic model does not depend on the Fourier number, while computational efficiency of the full 3D model drastically drops with increasing Fourier number. It makes the asymptotic model suitable for combined simulations of flow induced by the nucleus sublimation, where calculations of the body thermal state will take only a tiny fraction of overall computational cost.

The asymptotic model also provides a simple way to analyze the limit thermal regimes and to find bounds to the gas production rates. The knowing of these bounds is of special importance since the properties of comet nucleus material are known to date with large uncertainties

The asymptotic analysis is applied here to a knowingly simplified physical model of a comet nucleus. Most of these simplifications, however, are not obligatory, and the asymptotic analysis can be extended to more complex physical models. The model can be also adopted for non-spherical and non-homogeneous nuclei that undergo complex rotational motion.

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