

Fluid flow and dynamo action driven by differential rotation in a spherical shell

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Abstract

The interaction between shear flow and magnetic instabilities plays an important role for the dynamics of many celestial bodies ranging from galaxies to the dynamo regions of planets. Several laboratory and numerical experiments have been devoted to studying this interaction. Here we numerically explore the flow and dynamo action in a viscous and electrically conducting fluid between two differentially rotating spheres, the spherical Couette system. A laboratory realization of this setup has been build at the University of Maryland as a next generation dynamo experiment. Our simulations show a zoo of different flow instabilities. In particular the nearly two dimension quasi geostrophic solutions at larger outer boundary rotation rates are favorable for dynamo action. An extrapolation of the results suggest that the Maryland experiment could indeed yield dynamo action. When extrapolated to an imaginary planetary surface, the field is very axisymmetric and has a strong octupole component similar to the field observed on Saturn. We suggest that differential rotation driven by Helium precipitation may contribute to shaping the planets field.

1. Model

The fully 3d self-consistent MHD code MagIC was adapted by replacing the convective driving with imposed inner and outer boundary rotation rates. The system is solved in a frame of reference corotating with and is rule by four dimensionless parameters: the outer boundary rotation rate Ω , the differential rotation rate $\delta\Omega$, magnetic Prandtl number Pm , and the ratio of inner to outer sphere radius. We keep the latter fixed at 0.35 and vary the other three. We have chosen the viscous diffusion and the shell thickness to non-dimensionalize the problem so that Ω is the inverse of the Ekman number.

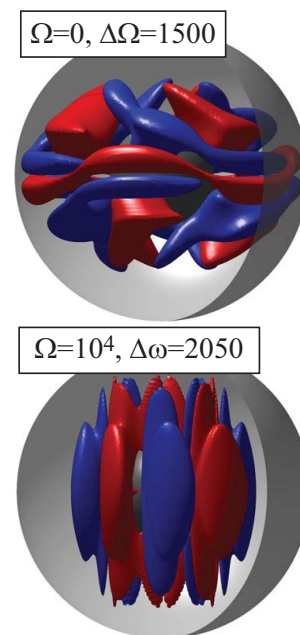


Figure 1: Visualization of the non-axisymmetric instability for slow (zero) outer boundary rotation (top) and fast outer boundary rotation (bottom) with iso-surfaces of outward (red) and inward (blue) non-axisymmetric radial velocity.

2. Flow instabilities

Depending on the outer boundary rotation rate Ω three different regimes can be distinguished. 1) For $\Omega < 1$ viscous forces prevail. The flow remains axisymmetric with a spherical symmetric angular velocity and a pair of northern and a southern meridional circulation cells as long as $\delta\Omega < \Omega$. As $\delta\Omega$ is increased, the meridional circulation cells start to advectively deform the flow and an equatorial jet develops. This eventually becomes unstable against a non-axisymmetric instability at some critical differ-

ential rotation rate $\delta\Omega_c$ (see Fig. 1). Super-rotation ($\delta\Omega > 0$) and sub-rotation ($\delta\Omega < 0$) cases show identical behavior and the instabilities are also independent of Ω . **2)** For intermediate values of Ω viscous and Coriolis effects compete. Super-rotation cases still yield equatorial jet type instabilities, but sub-rotation cases show a more complex meridional circulation pattern with two radially staggered sets of roles. This changes the type of the first non-axisymmetric instability. **3)** For $\Omega \geq 10^3$, Coriolis forces start to dominate. The flow assumes a quasi two-dimensional geostrophic structure [1] where the flow outside the cylinder tangent (TC) to the inner-boundary equator corotates with the outer boundary while the flow inside the cylinder predominantly assumes the intermediate angular velocity $\delta\Omega/2$. Meridional circulation is concentrated in jets at the TC with broader back flows inside the TC. A complex Stewartson boundary layer accommodates the differences angular velocity and hosts the meridional circulations jet. The layer becomes unstable against a non-axisymmetric instability that also assumes a rather geostrophic structure (see Fig. 1). Instability wave number and $|\delta\Omega_c|$ now depend on the sign of $\delta\Omega$ and also on Ω with $\delta\Omega_c \approx \Omega^{1/3}$ for $\delta\Omega > 0$ and $-\delta\Omega_c \approx \Omega^{0.54}$ for $\delta\Omega < 0$. However, the sub-rotation instability once more behaves like the super-rotation counterpart for $\Omega \geq 3 \times 10^{-6}$

3. Dynamo Action

We find that all non-axisymmetric flows sustain dynamo action provided the magnetic Prandtl number is large enough. Slow and intermediate outer boundary rotation rates are not very favorable to dynamo action and require larger magnetic Prandtl number. For $\Omega \geq 10^{-3}$ the critical magnetic Prandtl number Pm_c for dynamo action is roughly inversely proportional to Ω and at $\Omega = 10^{-4}$ we find $Pm_c = 5 \times 10^{-2}$. Simulations at $\Omega = 10^{-5}$ are ongoing but suggest an even smaller Pm_c . The Maryland dynamo experiment will run at $\Omega \approx 2.5 \times 10^7$ and $Pm \approx 10^{-5}$ where a simple extrapolation of our results suggest $Pm_c \approx 2 \times 10^{-5}$. Dynamo action thus seems within reach. Super-rotation seems to prefer dipolar magnetic fields (see Fig. 2) while sub-rotation prefers quadrupolar fields. At the outer boundary, the field is strongly concentrated to patches around the rotation axis. When extrapolated to an imaginary planetary surface, the dipolar fields are very axisymmetric and have a strong (positive) octupole component, similar to Saturn's magnetic field. We therefore suggest that differential rotation caused by Helium precipitation may contribute to shaping

Saturn's magnetic field.

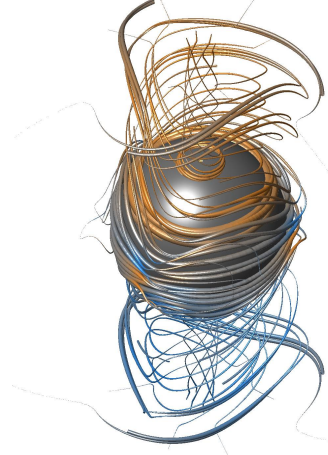


Figure 2: Dynamo solution at $\Omega = 10^4$, $\delta\Omega = 0.7$ and $Pm = 0.5$. Iso-surfaces of z-vorticity are shown in red and blue. The magnetic field lines are color-coded according to the direction of the radial field: red=outward, blue=inward. The lines thickness scales with the local magnetic energy.

4. Conclusion

The spherical Couette system shows a rich variety of different flow instabilities that all sustain dynamos. Increasing the outer boundary rotation rates seems to help dynamo action and allows to decrease the magnetic Prandtl number quite similar to what has been reported for convectively driven dynamo simulations [2]. Our results suggest that the Maryland Dynamo experiment may succeed, but further simulations at larger Ω values are required to establish this.

Acknowledgements

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References

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