

From Data To Equations: Inferring the Laws governing Saturn's Ring Temperature

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Abstract

Six years after Saturn Orbit Insertion (SOI), the Composite Infrared Spectrometer (CIRS) on-board the Cassini Spacecraft has been performing a thermal mapping of Saturn's main rings, by measuring the thermal radiance in the far-infrared ($[10-600] \text{ cm}^{-1}$) for different viewing geometries. So far, more than 2.5 millions individual spectra have been recorded, from Saturn's northern winter solstice till Saturn's northern spring. We present a first attempt of treating the data set globally by applying numerical data mining techniques inherited from the field of artificial intelligence, such as neural networks and genetic programming.

1. Introduction

Previous analysis has shown that the measured Saturn's main ring temperature is a non-linear function of multiple geometrical parameters, that represents the thermal response of Saturn Rings to time dependent illumination conditions and varying observation geometry. The different values measured for the temperature function contain mixed information on physical properties of the ring at different scales, from large scale reflecting collective thermal behavior of particles (mutual shadowing, mutual heating, heat transport through vertical motion), to microscopic scale (regolith thermal properties on individual particles surfaces).

Disentangling the physical implications of ring temperature variations is a major undertaking that has been attempted in the past years. Thermo-physical models have been developed [1], however generally constrained by subsets of measurements reflecting temperature variations with a limited number of geometrical parameters at a time. We propose in this work a global data mining approach of the data, related to Neural Networks and genetic programming.

2. Temperature statistical dependence upon geometrical space parameters

We investigated in the early stages of our work the true dimensionality of the data set by measuring the statistical dependence of temperature values upon each of the geometrical space parameters. To carry out this analysis, we have used the novel Hilbert-Schmidt Independence Criterion (HSIC), which is based on the eigenspectrum of covariance operators in reproducing kernel Hilbert spaces [2]. This method is able to capture any structure of dependence and hence used to construct a ranking of statistical dependence between measured temperatures and the parameters describing illumination and observation conditions.

3. Support Vector Machines approach

Given a data subset uses as training examples $D = \{\vec{x}_i : T_i\}_{i=0}^n$ (where x_i represent a set of geometrical parameters and T_i associated temperatures), a so-called 'supervised learning method' is able to extract regularities from the training data in order to predict the outcome of new combination of input parameters for which no measurements exist. *Support Vector Machines* (SVM) are state-of-art supervised methods of learning[3]. They have been shown to outperform in several scenarios other supervised learning methods like Neural Networks. An example of approximation of the temperature measurements obtained for the B ring is shown in Fig. 1, when SVM is trained with only one third of all available data. The other two third of the data are being predicted and match well the actual measurements. This method can be applied to predict future measurements or to extrapolate existing measurements such as to complete our multi-dimensional temperature mapping.

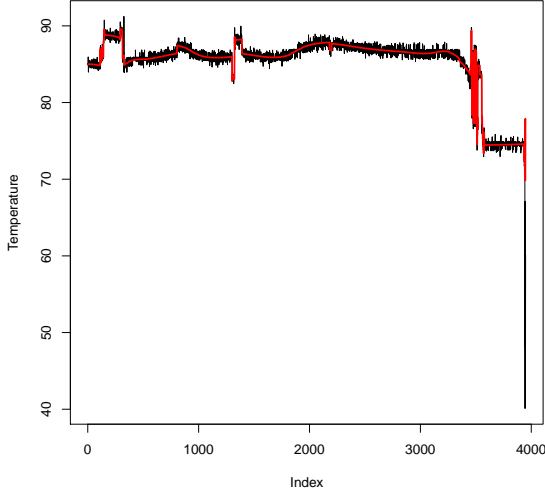


Figure 1: Approximation of Ring B temperature behavior when training data through a SVM

4. Genetic Programming approach

SVM have the disadvantage of not giving any mathematical laws for the temperature behavior as a function of the input parameters. For this reason, in order to support modeling work, we also focus on the search of relatively simple analytical description using a further machine learning method.

Genetic Programming methods try 'to breed' a population of formulas most efficient at evaluating a set of specified training data. A set of *building blocks* are used (sum, product, sine, exponentiation...) by the algorithm to obtain candidate temperature functions of the geometrical parameters that minimize an error function. Despite the simplicity of this family of algorithms, giving enough computational power, impressive results can be obtained, as shown on Fig. 2 where ring B temperatures are fitted using equation 1¹. Note that this formula is a very preliminary example to illustrate the genetic programming approach. It cannot have a physical sense, as no effort was made to ensure the dimensional homogeneity of the formula (the right side cannot be a temperature).

¹Where geometrical parameters are f as solar flux, s as spacecraft elevation, l as particle local time and a as the spacecraft-sun azimuth angle.

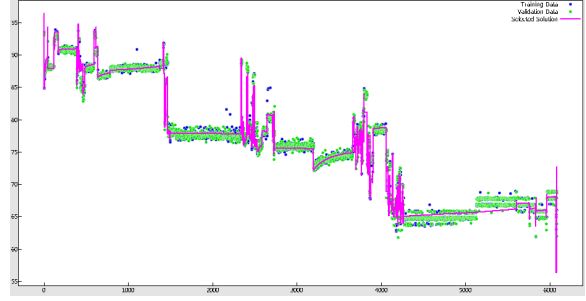


Figure 2: Approximation of ring B temperature measurements when training data using a genetic programming approach.

$$T_B(f, l, a, s) \sim \frac{76f}{\exp(s^{10} + a - 0.45) - \frac{4.36}{l} - f - 7.46} \quad (1)$$

5 Summary and Conclusions

This work-in-progress analysis looks promising for two main applications: 1- we support the physical modeling work by providing sets of mathematical functions, derived from the data, that characterize the system behavior without assuming separability of the input variables. 2- we predict the results one would obtain in regions of the observation geometrical space that can not be sampled due to experimental limitations (for example, because of the observational bias introduced by the orbit). In particular, the results of our regression analysis can be used to predict the outcome of future measurements, and hence, allow us to understand to what extent Saturn's rings thermal behavior is fully characterized ('predictable') by the existing measurements.

References

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