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PSTD-based Optimal Control Approach to the CONSERT Inverse Scattering Problem

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1. Introduction

The scope of CoNSERT, short for *Comet Nucleus Sounding Experiment by Radio Transmission* [1], is to perform a sounding of the comet 67P/C-G's core. This will be achieved by launching a lander, called *Philae*, onto the comet's surface. It will act as a receiver for the sounding signal transmitted by the Rosetta orbiter, and retransmit the received and processed signal. With these data, a three dimensional model of the comet's core (i.e., the material distribution with regard to the dielectric permittivity ε) is to be reconstructed.

2. Optimal Control Approach

An inverse problem can be formulated as optimization problem using an optimal control approach where the target functional $j(p)$ over the spatial domain Ω and time T can be expressed as:

$$j(p) = \frac{1}{2} \int_0^T \int_\Omega (u(p) - u_{obs}(p_u))^2 \, d\Omega \, dt \qquad (1)$$

$u(p)$ is a state, p the control parameter, and u^{obs} the observed state. We are trying to reconstruct the unknown parameter p_u such that $p = p_u$ if $j(p) = 0$. The constraining system of PDEs as well as the Lagrangian are derived in [2] resulting in the gradient necessary for the optimization:

$$\frac{\partial j}{\partial p} = \int_0^T \int_\Omega \lambda \frac{\partial D(p)}{\partial p} u(p) \, d\Omega \, dt \qquad (2)$$

where $D(p)$ denotes the differential operator and λ the adjoint state which is obtained by the adjoint differential operator $D^*(p)$.

The system of PDEs governing wave propagation in electromagnetic fields is defined by Maxwell's equations. With the permittivity ε as our target parameter and M discrete orbital observation points this leads to

$$j(\varepsilon) = \frac{\alpha}{2} \sum_{m=1}^{M} \int_0^T \int_\Omega (\beta E - \gamma E^{obs})^2 \delta_m \, d\Omega \, dt \qquad (3)$$

with the weights α_r, β_r and γ_r which can be used to adapt the target functional to the measured data. By applying the optimal control approach described in [3] we get the adjoint gradient, defined as:

$$\frac{\partial j}{\partial \varepsilon_r} = \int_0^T \int_\Omega E^* \frac{\partial E}{\partial t} d\Omega dt \qquad (4)$$

The weighted residual of E^{obs} and E serves as source for the adjoint system in the observation points M. E^* is the adjoint state obtained by applying the adjoint Maxwell's equations; the only difference to the non-adjoint equations is the change of the sign in the spatial derivatives. The PMLs necessary to truncate the computational domain are treated in the same manner [4]. The time-integration to compute the state E and adjoint state E^* is done in opposite direction. In case of isotropic materials ($\varepsilon_x = \varepsilon_y = \varepsilon_z$), only one field component is needed to compute the gradient. With this approach we derive an adjoint method for any time domain method to compute the direct as well as the adjoint fields. To ensure the correctness of gradients one can also derive the discretized adjoints and operators.

3. Algorithm and Implementation

The forward solver is invoked for the computation of the target functional and the gradient. We implemented a Pseudo-Spectral Time-Domain (PSTD) method based on a D-H formulation. In an FFT-based PSTD only two grid cells per wavelength are needed (according to Nyquist–Shannon sampling theorem) to compute the spatial derivatives. Due to numerical dispersion, an FDTD would need at least twelve cells per wavelength to achieve the same accuracy as a PSTD with an equal Courant–Friedrichs–Lewy (CFL) number.

However, there are some drawbacks. The PSTD is computationally more complex than the FDTD. Also, the maximal CFL number for the PSTD needs to be smaller than for the FDTD by at least a factor of $2/\pi$ [5] Since spatial and temporal unsteadiness lead to Gibbs phenomena and aliasing when computing the derivatives, the media in which the fields are propagating need to be smooth. Source definition is also an issue, which has to be treated in a special way. The source for the 1D and 2D case defined in [6] can easily be extended to three dimensions – Gibbs phenomena can be reduced significantly.

Despite these difficulties, a significant reduction in memory consumption compared to the FDTD leads to an increase of the computational domain from 8λ to 100λ on a single compute node, which outweighs all aforementioned drawbacks.

The adjoint solver is a slightly modified version of the forward solver in order to accomplish reverse time migration. The order in which the subroutines for D, E and H updates are called has to be the exact opposite of those in the forward solver in order to obtain correct gradients.

For the optimization step, we are using IpOpt [7], which offers the possibility to consider different constraints in the optimization process. These are given by theoretical comet models and assumptions regarding the inner composition as well as data from other instruments aboard Rosetta. With these constraints some local minima of the multimodal parameter space can be avoided [8].

4. Conclusions

Due to the use of the PSTD in the inversion process, we were able to increase the computational domain from 8λ to 100λ on a single compute node, and also to achieve an improved accuracy of the computation. With these improvements we are able to conduct baseband analyses of the whole comet or RF domain computations of parts of the comet (e.g., for the analysis of grazing incidence phenomena).

The introduction of the source consisting of eight Hertzian dipoles reduced the ripple (caused by Gibbs phenomena) below the magnitude of residual propagation and reflection within the PML.

The proposed algorithm to compute the gradient, as shown in Eq. 4, has proven to be fairly efficient.

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