



# Retaining the Primordial Cold Classical Kuiper Belt During a Transient Phase of Dynamical Instability

K. Batygin, M. E. Brown and W. C. Fraser

Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, CA 91125  
 (kbatygin@gps.caltech.edu)

## Abstract

The physical uniqueness of the cold classical population of the Kuiper belt suggests that its dynamical coherence has been maintained through out the solar system's lifetime. However, the overall lack of orbital excitation of the population has remained an elusive issue, especially in the context of a solar system formation model that is driven by a transient period of instability, where Neptune is temporarily eccentric. We argue that a locally formed cold belt can survive the instability, and its current dynamical structure can be reproduced. Using a simple analytical model, we show that comparatively fast apsidal precession and nodal recession of Neptune, during the eccentric phase, are essential for preservation of an unexcited state in the cold classical region. We confirm our results with self-consistent N-body simulations.

## 1. Introduction

The cold population is distinctive from the rest of the Kuiper belt in its orbital and physical characteristics. First and foremost, as its name suggests the orbital distribution of the cold belt is dynamically unexcited, with inclinations not exceeding  $\sim 5$  deg [3]. A second distinction is the exclusively red colors of cold classical KBOs [13]. In a similar manner, the size distribution of the cold population differs significantly from that of the hot classical population [4]. Finally, the fraction of wide binaries present in the cold population is uniquely large, suggesting that the population never experienced close encounters with Neptune [11].

The formation mechanism for the cold classical population poses a considerable theoretical challenge. Within the context of the "NICE" model [14], where Neptune scatters onto its current orbit abruptly with a high eccentricity, [5] advocated an emplacement scenario for the cold classicals. However, the cold population that is produced in such simulations is not cold enough and not physically distinct from the hot popu-

lations. As a result, no coherent picture of the formation of the cold classical population exists.

## 2. Secular Excitation of the Cold Kuiper-Belt

Because the majority of the objects in the cold belt do not reside in low-order mean motion resonances with Neptune, the interactions between the two are primarily secular in nature. This allows one to construct a simple analytical model for the excitation of the cold belt. The scenario we consider is one where Neptune is scattered onto its current semi-major axis with a high eccentricity  $e_0$  and inclination  $i_0$ . This is followed by circularization of the orbit due to dynamical friction, which can be approximated as exponential decay of eccentricity and inclination on timescales  $\tau_e$  and  $\tau_i$  respectively. In terms of complex Poincaré variables ( $x = e \exp(i\varpi)$ ,  $y = i \exp(i\Omega)$ ), we can formulate the first-order Lagrange's equations for Neptune as follows:

$$\frac{dx_n}{dt} = igx_n - \frac{x_n}{\tau_e} \quad \frac{dy_n}{dt} = ify_n - \frac{y_n}{\tau_i}, \quad (1)$$

where  $g = \langle \dot{\varpi}_N \rangle$  and  $f = \langle \dot{\Omega}_N \rangle$  are the average precession rates of the longitude of perihelion and ascending node.

Similarly, retaining only secular terms up to second order in eccentricity and inclination in the disturbing potential, the equations of motion for a cold Kuiper belt object read

$$\frac{dx_{kbo}}{dt} = iAx_{kbo} + iA_nx_n \quad \frac{dy_{kbo}}{dt} = iBy_{kbo} + iB_ny_n, \quad (2)$$

where  $A$ ,  $A_n$ ,  $B$ , and  $B_n$  are constants that depend only on the planetary masses and semi-major axes ratios of Neptune to KBOs (e.g. Ch.7 of [10]). Prior to the scattering event, Neptune is sufficiently far away from the Kuiper belt for the interactions to be neglected. Thus, the initial conditions of the KBO can

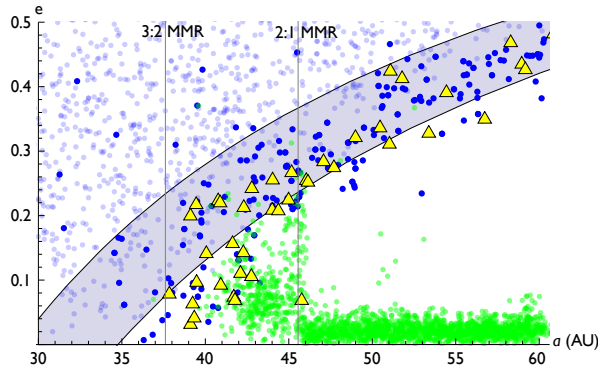


Figure 1: Eccentricity distribution of the remnant planetesimal disk of a NICE-model type N-body simulation that contains a cold primordial belt. The pale blue dots show objects that originated interior to  $\sim 35$  AU, 20 Myr after the instability. The dark blue dots represent objects that originated interior to 35 AU, but are stable over 500 Myr. Green dots represent the test-particles that originate between 40 & 60 AU. Yellow triangles represent test particles that originated between 35 and 40 AU. The inclinations of objects in the cold classical region (i.e. green dots) do not exceed  $i_{max} < 4$  deg.

be taken to be  $([x, y] = \vec{0})$ . This system of equations can be solved in closed form. Furthermore, under suitable approximations, it can be shown (see [1] for full derivation) that the final orbits (after Neptune’s orbit has circularized) of KBOs take the form:

$$e_{kbo}^{final} \simeq e_n^0 \frac{A_n}{g - A} \quad i_{kbo}^{final} \simeq i_n^0 \frac{B_n}{f - B}. \quad (3)$$

This solution implies that dynamically cold orbits can be retained if the magnitude of Neptune’s apsidal precession and nodal recession rates ( $g$  and  $f$ ), during its eccentric phase, significantly exceeded that of the free apsidal precession and nodal recession rates ( $A$  and  $B$ ) of the KBOs. Quantitatively, it is sufficient for  $g$  and  $f$  to exceed the  $g_8$  and  $f_8$  eigenfrequencies of the Laplace-Lagrange secular solution by a factor of  $\sim$  few.

We have confirmed our results with a series of self-consistent N-body simulations, similar in form to the NICE model. A synthetic Kuiper belt, constructed in one of the simulations is shown in Figures 1.

### 3. Summary and Conclusions

Here, we have argued that in-situ formation of the cold classical population of the Kuiper belt is fully consistent with an instability-driven formation model for the

solar system. Particularly, we have shown, from simple analytical considerations, that the cold belt can survive the transient period of dynamical instability inherent to the planets, if the average apsidal precession and nodal recession rates of Neptune exceeded what is observed in today’s solar system. Numerical N-body integrations confirm the results of the analytical calculations and thus suggest that both, the hot and cold populations of the Kuiper belt can be successfully reproduced. The dynamical evolution of cold classicals we propose here is in line with the uniqueness of cold classical’s physical characteristics.

### Acknowledgements

We thank Alessandro Morbidelli, Hal Levison, and Peter Goldreich for useful conversations.

### References

- [1] Batygin, K., Brown, M. E., Fraser, W. C. 2011, ApJ, submitted
- [2] Brown, M. E. 2001, Icarus, 151, 190
- [3] Fraser, W. C., Brown, M. E., & Schwamb, M. E. 2010, Icarus, 210, 944
- [4] Levison, H. F., Morbidelli, A., Vanlaerhoven, C., Gomes, R., & Tsiganis, K. 2008, Icarus, 196, 258
- [5] Morbidelli, A., Levison, H. F., & Gomes, R. 2008, The Solar System Beyond Neptune, 275
- [6] Morbidelli, A., Brasser, R., Tsiganis, K., Gomes, R., & Levison, H. F. 2009, A&A, 507, 1041
- [7] Murray, C. D., & Dermott, S. F. 1999, Solar system dynamics by Murray, C. D., 1999
- [8] Parker, A. H., & Kavelaars, J. J. 2010, ApJL, 722, L204
- [9] Trujillo, C. A., & Brown, M. E. 2002, ApJL, 566, L125
- [10] Tsiganis, K., Gomes, R., Morbidelli, A., & Levison, H. F. 2005, Nature, 435, 459