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The Dynamical Environment of an Elongated Body Application to the Restricted Full Two Body Problem

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Abstract

The problem of a massless particle orbiting a non-spherical body is described by computing an approximation of the gravitational potential and using the equations of motion of the Restricted Full Two Body Problem, RF2BP. The gravitational potential is computed by expanding it in terms of spherical Bessel functions and spherical harmonics and by matching this expression with the well known MacCullagh's formula. Therefore, we have developed an expression for the approximate potential of a body, of whom only the mass and moments of inertia are known, which is valid near the surface of the body, as well as, far away from it. This potential is then used to study the dynamical environment of a non-spherical body and how the stable and unstable manifolds of periodic orbits about Lagrange points, or about the entire body, affect the environment.

1 Introduction

In the last few years, we have witnessed several missions sent to asteroids and comets, and more are planned for the future. Hence, understanding the dynamical environment of the small bodies of the Solar System is a key factor that will allow us to design missions to them. However, the problem faced when targeting these bodies is the lack of knowledge about their shape and gravitational potential. In this paper, we have developed a second order expansion that can be used as a first approximation of the potential for any celestial body of whom the mass and moments of inertia are known. This expansion satisfies Poisson's equation, inside the sphere circumscribing the body, and Laplace's equation outside.

The environment of an elongated rotating body is very rich in dynamics. This is because the relative equilibrium points can have periodic orbits with stable and unstable manifolds that intersect the body, depending on the system parameters. In this paper, we have analysed the role that these manifolds have in the dynamics around rotating non-spherical bodies using the gravitational potential previously developed.

2 The gravitational potential

The gravitational potential, Φ, satisfies Poisson's equation at points inside the body, but at points external to the body, Poisson's equation reduces to Laplace's equation as the density of the body vanishes. To solve for the potential of a particular mass distribution we solve independently both equations and impose that the solution and the derivative match at a given spherical boundary: $r = R$. Using spherical harmonics and spherical Bessel functions the internal potential is [4]

$$\Phi(r,\theta,\varphi) = \frac{G}{R}\sum_{l=0}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{n} j_n\left(\frac{\alpha_{ln}r}{R}\right)P_n^m(\cos\theta)\cdot \cdot\left(A_{lnm}\cos(m\varphi)+B_{lnm}\sin(m\varphi)\right), \quad (1)$$

which gives us an analytical density function $\rho(r,\theta,\varphi)$ in terms of the same functions.

2.1 The choice of coefficients

Since the density of the body is not known, a) we have infinite coefficients b) just a few constraints: the internal and external potential have to match smoothly at $r = R$, $\lim_{r\to R}\rho = 0$, and the total mass and moments of inertia are given. Now, we can choose the coefficients in such a way that they satisfy the constraints. Changing the coefficients changes the density of the body, and therefore the dynamics near the surface, but the approximation is still valid as it matches smoothly with the external potential.

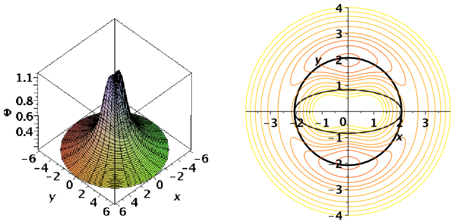

Figure 1: Potential of asteroid 1620 Geographos in non-dimensional coordinates and zero velocity curves using data from [3].

3 Dynamics

For a symmetric second order expansion of the potential, there can exist two equilibrium points (external to the body) aligned with the longest axis and two with the shortest axis. The long axis equilibrium points always have a saddle-centre behaviour for all possible moments of inertia of the body and angular velocity, whereas the short axis ones undergo a Hopf bifurcation.

The stable and unstable manifolds of the periodic orbits about the long axis equilibrium can come from or crash onto the elongated body depending on the system parameters. Therefore, these tubes form paths that particles such as dust, little rocks or spacecraft can follow to land or depart from the body. Furthermore, the stable manifolds of unstable periodic orbits that come from the elongated body, divide the surface of the body in different regions depending on where the particle has come from (angle ϕ) and what was the angle of the velocity (α) relative to the tangent at angle ϕ.

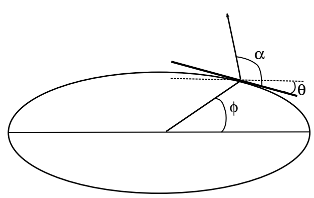

Figure 2: Angles on the surface of the body, $\phi \in [0, 179]$ and $\alpha \in [0, 180]$.

4. Summary and Conclusions

We have developed an approximation of the gravitational potential of a non-spherical body, in terms of its mass and moments of inertia that is valid near the surface and far away from it. With this potential, the

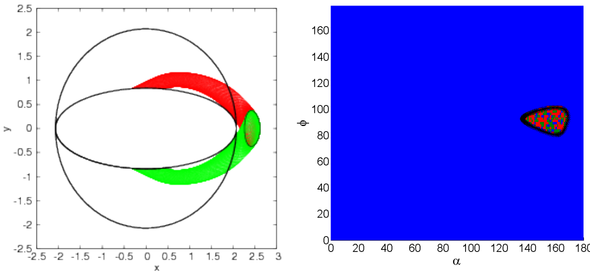

Figure 3: Left: Stable (green) and unstable (red) manifold for a periodic orbit around the long axis equilibria of Geographos in non-dimensional, rotating coordinates. Right: initial conditions with the same energy on Geographos surface that crash back (blue), escape (red), or still orbit (green), and the initial conditions of stable manifold of the periodic orbit (black).

stable and unstable manifolds of periodic orbits can be followed near the surface of the body. These manifolds near the surface give us information about the dynamical environment of the rotating elongated body, such as where there is focusing of material, redistribution of mass or fuel-optimal oncoming trajectories to the body.

Acknowledgements

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