

Angular momentum exchange during secular migration of two - planet systems

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Abstract

We investigate the secular dynamics of two-planet coplanar systems evolving under mutual gravitational interactions and dissipative forces. We consider two mechanisms responsible for the planetary migration: star-planet (or planet-satellite) tidal interactions and interactions of a planet with a gaseous disc. We show that each migration mechanism is characterized by a specific law of orbital angular momentum exchange. Calculating stationary solutions of the conservative secular problem and taking into account the orbital angular momentum leakage, we trace the evolutionary routes followed by the planet pairs during the migration process. This procedure allows us to recover the dynamical history of two-planet systems and constrain parameters of the involved physical processes.

1. Introduction

Dissipative interactions remove (or add) orbital energy from the planetary system, producing changes of semi-major axes. Angular momentum is also exchanged between the orbits and the exterior components. In [3], we have purposed a generic approach to describe the exchange of the orbital angular momentum of the system during migration.

2. Angular momentum exchange

We consider a three-body system consisting of a central star with mass m_0 and two coplanar planets with masses m_1 and m_2 . We assume that the system is far away from any mean-motion resonance. If dissipation is sufficiently slow the long-term variations of the orbital elements can be separated into two components: secular and external

$$\dot{a}_i = \dot{a}_i^{\text{sec}} + \dot{a}_i^{\text{ext}} \quad ; \quad \dot{e}_i = \dot{e}_i^{\text{sec}} + \dot{e}_i^{\text{ext}}. \quad (1)$$

Taking derivative of the orbital angular momentum of the system and identifying both components we have

$$\dot{e}_i^{\text{sec}} = -\frac{m'_j}{m'_i} \sqrt{\frac{a_j}{a_i}} \frac{e_j}{e_i} \sqrt{\frac{1-e_i^2}{1-e_j^2}} \dot{e}_j^{\text{sec}}, \quad (2)$$

$$\dot{e}_i^{\text{ext}} = (1-\alpha) \frac{(1-e_i^2)}{2a_i e_i} \dot{a}_i, \quad e_i \neq 0, \quad (3)$$

where

$$\alpha = 1 - \mathcal{F}(e_i, \mathbf{par}) \frac{2e_i^2}{1-e_i^2}, \quad (4)$$

with $\mathcal{F}(e_i, \mathbf{par})$ being a characteristic function of the migration process, defined through the condition

$$\frac{\dot{e}_i^{\text{ext}}}{e_i} = \mathcal{F}(e_i, \mathbf{par}) \frac{\dot{a}_i}{a_i}, \quad (5)$$

where the vector \mathbf{par} is composed of physical parameters of the process under study. The function α can thus be used as a convenient measure of the non-conservation of the orbital angular momentum.

3. Example: tidal interactions

We consider a short-period planet orbiting a slow-rotating central star ($\Omega_0 \ll n_1$), assuming that tidal interaction does not affect the outer planet. Through the mean variations of the orbital elements (see [1]), using a linear tidal model ([4]), we have

$$\alpha = 1 - \frac{(9+7D)e_1^2}{(1-e_1^2)[1+(23+7D)e_1^2]}. \quad (6)$$

$D = (Q'_0/Q'_1)(m_0/m_1)^2(R_1/R_0)^5$ is the parameter of the problem, where Q_i and R_i are dissipation functions and radii, respectively. It can be shown that $\alpha \propto -\dot{\Omega}_0/\dot{a}_1$, hence, since $\dot{a}_1 < 0$ due to tides and $\alpha > 0$ during orbital decay of the inner planet (see Fig. 1), it results in $\dot{\Omega}_0 > 0$. The portion $1-\alpha$ is absorbed by the system, damping the eccentricity of the inner planet, whereas α is transferred to the central star and accelerates the stellar rotation

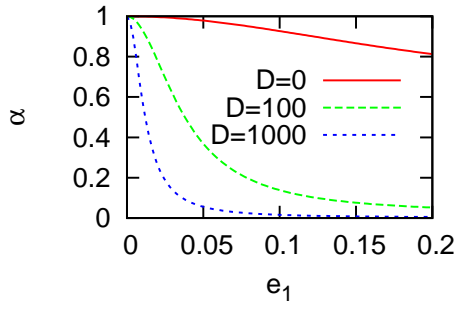


Figure 1: α as a function of e_1 , in the case of a close-in planet tidal evolution.

3.1. Evolutionary routes

Stationary solutions of the secular problem (or equilibrium values of e_1, e_2) are uniquely determined by masses, semi-major axes and orbital angular momentum. When migration is taken into account, a_1 is modified by Δa_1 and the orbital angular momentum of the system is then corrected by the amount $\alpha(L_1/2a_1)\Delta a_1$, where $L_1 \propto \sqrt{a_1(1-e_1^2)}$. A curve of stationary solutions is obtained in the space $(e_i, n_1/n_2)$ for the whole range of a_1 . Fig 2 shows the possible tracks of the CoRoT-7 two-planet system ([2]). The migration paths are sensible to the values of D , and consequently, to the orbital angular momentum exchange defined by the function α , according to Eq. (6).

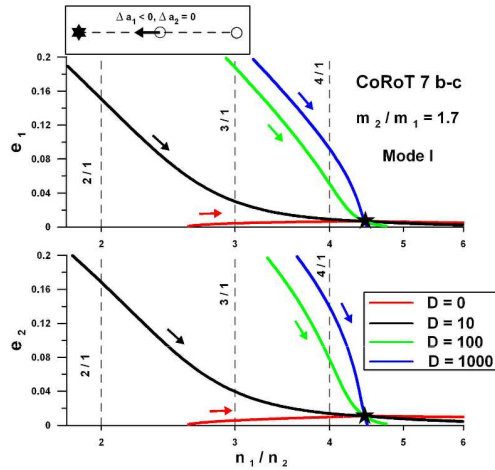


Figure 2: Migration paths of CoRoT-7 system parameterized by constant values of D . The box at the top shows the schematic view of the migration scenario.

4. Summary and Conclusions

In this work we model analytically the orbital angular momentum exchange of two-planet coplanar systems evolving under dissipative forces. We have shown that the angular momentum exchange can be calculated through the α -function. For each dissipative mechanism considered, α was calculated as a function of the planet eccentricity of the migrating planet and physical parameters involved in the process. The development of the α -function for each dissipative process, and the consequent calculation of evolutionary routes allow us to reassemble the starting configurations and migration history of the planet systems on the basis of their current orbital configurations. In addition, the analysis of the orbital angular momentum evolution during migration of the system allows us to constrain parameters of the involved dissipative physical processes.

Acknowledgements

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References

- [1] Ferraz-Mello, S., Rodríguez, A., and Hussmann, H.: Tidal friction in close-in satellites and exoplanets: The Darwin theory re-visited. *Celest. Mech. Dyn. Astr.*, Vol. 101, pp. 171-201, 2008.
- [2] Léger, A. et al.: Transiting exoplanets from the CoRoT space mission. VIII. CoRoT-7b: the first super-Earth with measured radius, *A&A*, Vol. 506, pp. 287-302, 2009.
- [3] Michtchenko, T. A. and Rodríguez, A.: Modeling the secular evolution of migrating planet pairs, (*MNRAS*, in press, preprint: arXiv:1103.5485) 2011.
- [4] Mignard, F.: The evolution of the lunar orbit revisited - I, *Moon and Planets*, Vol. 20, pp. 301-315, 1979.