Stationary equatorial modes in Venus’s lower atmosphere

Takeshi Imamura
Institute of Space and Astronautical Science, Japan Aerospace Exploration Agency, 3-1-1, Yoshinodai, Sagamihara, Kanagawa 252-5210, Japan (imamura.takeshi@jaxa.jp)

Abstract
Possible existence of heat-induced circulation in Venus’s tropical atmosphere is studied using an analytical solution on the equatorial $\beta$-plane. Large-scale topographic rises including Aphrodite terra in the equatorial region provide heat sources in the atmosphere away from the surface. Such heat sources might induce planetary-scale axi-asymmetric tropical circulation. Although the (retrograde) rotation of Venus is slow, the low static stability of the atmosphere leads to a relatively small deformation radius, enabling the circulation being trapped in the low latitude. The response of the circulation is amplified when the background wind speed is near the phase speed of the Rossby wave or the inertia-gravity wave. The correlation between the zonal and meridional winds induces equatorward transport of zonal momentum and might contribute to the maintenance of the super-rotation. The combination of the rising motion in the heating region and the descending motion in the cooling region transports heat upward, thereby stabilizing the stratification.

1. Introduction
The motions of the lower atmosphere of Venus are not well understood. Entry probes revealed that the speed of the super-rotation decreases with depth and becomes weaker than 1 m s$^{-1}$ near the surface. The mean meridional circulation and eddy motions are totally unknown. The dynamical state of the lower atmosphere might be important in the maintenance of the super-rotation, whose mechanism is a long-standing mystery.

The temperature lapse rate below clouds is very close to dry adiabat but is slightly subadiabatic. Most of the radiative transfer calculations predict unstable stratification in radiative equilibrium and expect convective adjustment, which leads to neutral stability. Heat transport by large-scale circulation might explain the observed stratification.

The present study addresses the possibility of forced circulations in Venus’s lower atmosphere and their roles in the momentum and energy balance using an analytical technique. Highlands in the Venus’s equatorial region may serve as a heat source in the atmosphere away from the surface, thereby inducing axi-asymmetric stationary circulation. This situation is analogous to the role of Tibetan Plateau in generating a high pressure in Earth’s atmosphere.

2. Method of linear solution
Matsuno [1] obtained solutions of forced stationary motions together with free waves in the equatorial region. The theory is based on the equatorial $\beta$-plane approximation and the separation of variables into vertical and horizontal structures. The background atmosphere was assumed to be at rest. Here the Matsuno’s method is extended to non-zero background wind.

Since the rotation speed of Venus is only 1.6 m s$^{-1}$, the role of the super-rotational background flow might be important in providing a rotational frame for eddies embedded in the background flow. Observations suggest that the super-rotation in Venus’s lower atmosphere is well approximated by solid-body rotation. Thus, it is reasonable to describe atmospheric motions in such a rotational frame moving with the background flow. Then the governing equations for the horizontal structure in a non-dimensionalized form are:

\begin{align}
-\omega u - yv + ik\Phi + \alpha u &= 0, \quad (1) \\
-\omega v + yu + \partial\Phi/\partial y + \alpha v &= 0, \quad (2) \\
-\omega\Phi + iku + \partial\Phi/\partial y + \alpha\Phi &= Q, \quad (3)
\end{align}

where $u$ and $v$ are the eddy zonal and meridional velocities, respectively, $\Phi$ is the eddy geopotential, $k$ is the horizontal wavenumber, $\omega = -ku$ is the doppler shift with $u$ being the background wind speed, $y$ is the northward distance, $\alpha$ is the dissipation rate representing Rayleigh friction and Newtonian cooling, and $Q$ is the forcing term which is proportional to the vertical derivative of the diabatic heating. The $\beta$ value...
used in the non-dimensionalization is defined for the coordinate system rotating with \( \bar{u} \). Field variables \( u, v, \Phi \) and \( Q \) depend on the vertical coordinate \( z \) as \( \cos(2\pi z/\lambda_z) \) with vertical boundaries placed at \( z = 0 \) and \( \lambda_z/2 \). The eddy vertical velocity and the diabatic heating depends on \( z \) as \( \sin(2\pi z/\lambda_z) \). The boundary conditions to solve these equations are \((u, v, \Phi) \rightarrow 0\) when \( y \rightarrow \pm\infty \).

The free wave solutions (\( \alpha = 0 \) and \( Q = 0 \)) constitute a set of eigenfunctions forming a complete set, and thus any arbitrary set of three functions \( u, v \) and \( \Phi \) can be expressed by a linear combination of the eigenfunctions. Once the set of forcing functions for \( u, v \) and \( \Phi \), namely \((0, 0, Q)\), are given by a linear combination of the eigenfunctions, it is straightforward to calculate the responses of \( u, v \) and \( \Phi \) to the forcing.

Here we seek for the solution caused by a forcing of the form

\[
Q \propto \exp(-y^2/2).
\]

In this case the series of eigenfunctions forming the solution terminates up to the forth term: Kelvin wave \((n = -1)\), eastward and westward inertia-gravity waves \((n = 1)\), and Rossby wave \((n = 1)\). The excited modes will have large amplitudes when \( \omega \) is close to one of the eigenfrequencies of these four modes, because such a mode is nearly resonant to the excitation.

3. Forcing in the Venus atmosphere

The height of the topographic rises in the equatorial region is \( \sim 6 \) km. In this region the incoming solar energy will be transferred to the atmosphere at this height or above. Modeling the exact form of this anomalous thermal forcing requires information on the vertical heat transport from the surface via radiation and diffusion and the horizontal heat transport from higher latitudes via meridional circulation and horizontal mixing. Instead of a detailed analysis of the thermal forcing, we tentatively assume that a quarter of the incoming solar energy in the equatorial region \((10 \text{ W m}^{-2})\) is distributed to an atmospheric layer with a thickness of 10 km to give the maximum amplitude of the forcing. The vertical wavelength of the forcing function and the forced circulation is assumed to be 10 km. The zonal wavenumber is assumed to be unity.

Since the height region where the circulation is induced is uncertain, the background wind speed and the static stability are considered as free parameters. The typical values will be \( \bar{u} = 0-10 \text{ m s}^{-1} \) and \( dT/dz - \Gamma = 0.01-1 \text{ K km}^{-1} \).

The dissipation rate is also uncertain. The radiative relaxation time is considered to be longer than 10 years. The diffusion time is similar when the eddy diffusion coefficient is \( \sim 0.1 \text{ m}^2 \text{ s}^{-1} \). Here the dissipation time \( \tau \) is assumed to be 1–10 years.

4. Solution of the circulation

The solution is sensitive especially to \( \bar{u} \); the response becomes large when the Doppler shift coincides with the frequency of the Rossby wave or the eastward inertia-gravity wave. An example of the solutions is shown in Fig. 1. The parameters adopted are; \( \bar{u} = 0.5 \text{ m s}^{-1} \) which is near resonant to the Rossby wave, \( dT/dz - \Gamma = 0.1 \text{ K km}^{-1} \), and \( \tau = 3 \) years. In this case the deformation radius (length unit for normalization) is 3780 km. The calculated horizontal convergence of the zonal momentum is \( 0.023 \text{ m s}^{-1} \text{ day}^{-1} \).

The dependences of the solution on all parameters and the implications for the energy and momentum transport will be reported elsewhere.

Figure 1: An example of the horizontal structure of the forced circulation. The length unit is the deformation radius of 3780 km. The unit length for the wind vectors (arrows) is 10 m s\(^{-1}\).

References