



Interior Structure, Mantle Rheology, and Tidal Heating of Rocky super-Earths

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1. Introduction

The confirmed exoplanets CoRoT-7b and Kepler-10b have motivated us to investigate their interior structure, thermal state, and tidal evolution. Comparable large mean densities ρ_p of (9.7 ± 1.9) for CoRoT-7b and (8.8 ± 2.5) Mg m^{-3} for Kepler-10b suggest that both planets have a rocky bulk composition. Both planets are expected to be in synchronous rotation and subject to tidal forces due to proximity to their primaries [1].

2. Model

2.1 Interior structure

We consider a one-dimensional, four-layer structural model consisting of an iron core overlain by three silicate mantle shells representing different mineralogical phases of MgSiO_3 . The interior structure is then obtained by solving the mass and energy balance equations in conjunction with an equation of state (EoS) for the radial density distribution [2]. Due to its ability to facilitate extrapolation to exceptionally high pressures, we have implemented the semi-empirical generalized Rydberg EoS, which is compliant with recent first principals calculations. To model self-consistently the present thermal state of planetary mantles, we adopt a mixing length formulation [3]. The idea behind this concept is that internally generated heat is transferred by vertical motion of fluid parcels, which will lose their individuality after migrating across characteristic length scales. The feasibility of this approach was demonstrated in various studies [e.g.,4].

2.2 Tidal heating

The tidally dissipated energy H within a synchronously rotating planet or satellite strongly depends on its mean motion n , orbital eccentricity e , and planetary radius R_p according to [5]

$$H = -\frac{21}{2} \frac{n^5 e^2 R_p^5}{G} \text{Im}(k_2),$$

where G is the gravitational constant and k_2 is the degree-2 tidal potential Love number.

2.3 Rheological properties

The tidal heating rate H is dependent on the material properties inside the planet, which are globally described by k_2 . We model the viscosity η in the framework of a temperature(T)- and pressure(P)-dependent Arrhenius law, considering vacancy diffusion as the predominant creep activation mechanism in rocky exoplanet mantles [e.g.,6]

$$\eta = \eta_{ref} \exp\left[\frac{\Delta Q}{RT}\right].$$

Where η_{ref} is a reference viscosity and R is the universal gas constant. $\Delta Q = \Delta E + P\Delta V$ is the creep activation enthalpy with ΔE the activation energy and ΔV the activation volume. We approximate the pressure-induced reduction of the creep activation volume ΔV with depth by applying a model, in which the vacancies are represented as spherical cavities within an elastic continuum [7].

The elastic rigidity or shear modulus is calculated according to the generalized Guinan-Steinberg formula [8]

$$\mu = \mu_0 + \mu_0' P \left(\frac{a_1}{x^{1/3}} + \frac{a_2}{x^{2/3}} + \frac{a_3}{x} \right).$$

Here, x is the compression ratio ρ/ρ_0 , and the subscript 0 refers to zero pressure. The coefficients a_1 , a_2 , and a_3 can be obtained either through a fit to data from high-pressure experiments or based on theoretical considerations. This formulation of the

rigidity is complementary to high-pressure EoS, thereby providing accurate shear moduli when extrapolated to high pressures.

3. Results

Our calculated results for CoRoT-7b and Kepler-10b are collected in Tab. 1. If compared to the Earth, both planets are enriched in heavy elements as implied by the relatively high iron core mass fraction. When CoRoT-7b is compared to Kepler-10b, its larger mean density yields a somewhat larger mantle thickness and generally higher temperatures and pressures.

Table 1: Modeling results for CoRoT-7b and Kepler-10b.

Planet	CoRoT-7b	Kepler-10b	Earth [A,B,C,D]
cmf [wt.-%]	60	59.5	32.6
D_m [km]	3330	2950	2890
T_{cmb} [K]	5580	5060	3740
T_c [K]	10,100	8960	5030
q_s [mW m ⁻²]	123	106	65
q_{cmb} [mW m ⁻²]	58.6	57.2	20
P_{cmb} [GPa]	615	410	136
P_c [GPa]	3410	2230	364
g_s [m s ⁻²]	27.3	22.4	9.83

We find that thermal conductivity and density increase with depth by a factor of two, however, thermal expansivity decreases by more than an order of magnitude across the mantle of both planets.

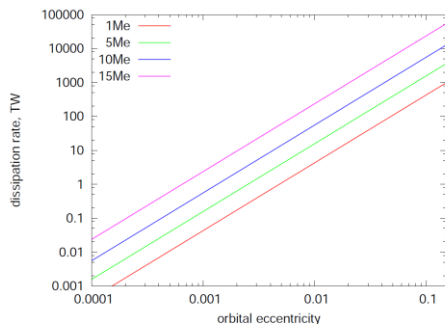


Figure 1: Tidal dissipation rate as a function of orbital eccentricity and planetary mass in units of Earth masses.

Figure 1 illustrates that tidally-induced, time-variable surface distortions and gravity variations due to tiny radial and librational tides may occur along slightly eccentric orbits, thereby causing the dissipation of tidal energy. This effect will strengthen with increasing planetary mass due to larger dissipative

volumes. Therefore, tidal heating is a viable present-day heat source for close-in exoplanets, other than heating through decay of long-lived radioactive elements such as U, Th, and K.

4. Conclusions

The calculations show that both CoRoT-7b and Kepler-10b have a bulk composition similar to Mercury (i.e., enriched in heavy elements like iron). The body tide Love numbers strongly scale with total mass. If CoRoT-7b and Kepler-10b are in resonant and sufficiently eccentric orbits, tidal heating would substantially affect their present thermal states and orbital evolution.

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