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Secular evolution of the orbit distance and asteroid hazard

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Abstract

We introduce two methods to estimate an interval J of possible future crossing times between the orbit of an asteroid and the one of the Earth. They are based on the computation of the secular evolution of the orbit distance [4]. With the results of this work we can compile a priority list of asteroids to be processed to assess their collision risk with our planet.

1. Introduction

The orbit distance between the orbit of an asteroid and the orbit of the Earth, called MOID in the literature, is a useful tool to investigate the asteroid hazard. We shall call d_{min} this distance. There are several algorithms available to compute d_{min}, e.g. [6], [2].

Given an orbit E of an asteroid at time t_0, with its covariance matrix Γ_E, we investigate possible methods to estimate the evolution of the orbit distance, with its uncertainty, and to define an interval J of times at which the orbit of the asteroid may cross the one of the Earth ($d_{min} = 0$). For this purpose we use different analytical tools developed at the University of Pisa in the last decade, see [1, 2, 3, 4, 5].

2. Uncertainty of the orbit distance

The constraint $d_{min} \geq 0$ produces a singularity in the orbit distance evolution at crossing times. We can solve this problem by giving in a suitable way a positive or negative sign to d_{min}. We denote by $\tilde{d}_{min}$ this '*orbit distance with sign*', where the sign is chosen according to a rule sketched in Fig. 1. Let τ_1, τ_2 be the tangent vectors to the orbits at their mutually closest points and denote by Δ_{min} the line joining these points ($|\Delta_{min}| = d_{min}$). We choose $\tilde{d}_{min} = d_{min}$ if $\tau_1 \times \tau_2$ and Δ_{min} have the same orientation, $\tilde{d}_{min} = -d_{min}$ in the other case, see [3].

The uncertainty of $\tilde{d}_{min}$ is well defined (as opposite

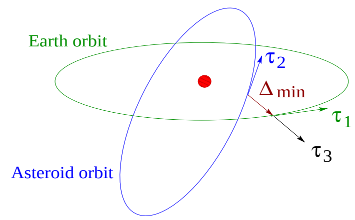

Figure 1: Giving a sign to d_{min}.

to d_{min}) and is given by

$$\Gamma_{\tilde{d}_{min}} = \frac{\partial \tilde{d}_{min}}{\partial E} \Gamma_E \left[\frac{\partial \tilde{d}_{min}}{\partial E} \right]^T . \qquad (1)$$

3. Secular evolution of planet-crossing orbits

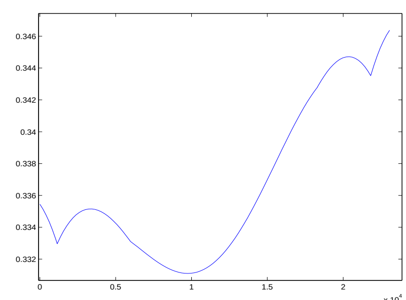

Figure 2: Secular evolution of the eccentricity for (1620) Geographos.

In [1] an analytical procedure has been introduced to apply the averaging principle to planet-crossing orbits. In this way we can compute the secular evolutions of near-Earth asteroids, whose orbits usually cross the orbit of the Earth during their evolution. However the solutions of the averaged equations are singular at crossing times, see the corners in Fig. 2.

4. Evolution of $\tilde{d}_{min}$

Surprisingly enough, the secular evolution of $\tilde{d}_{min}$ is regular even at crossings [4]: the tangent line exists also for $\tilde{d}_{min} = 0$. Thus we can consider the linearization of such evolution. In Fig.3 we draw the ordinary (blue), secular (red) and secular linearized (dotted) evolution of $\tilde{d}_{min}$ for the asteroid 1979 XB.

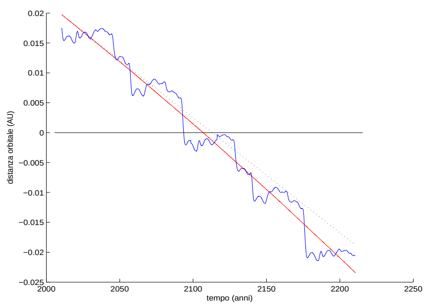

Figure 3: Evolution of $\tilde{d}_{min}$ for 1979 XB.

5. Uncertainty propagation of $\tilde{d}_{min}$

We propose two methods to compute the interval J.

First method. We sample the *line of variations* [5], which is a sort of '*spine*' of the confidence region, and compute the orbit distance $\tilde{d}_{min}$ for each virtual asteroid (VA) of the sample. Then we compute the time derivative of $\tilde{d}_{min}$ for each VA and extrapolate the crossing times by a linear approximation of the evolution. We set $J = [t_1, t_2]$, with t_1, t_2 the minimum and maximum crossing times obtained (see Fig. 4(a)).

Second method. The secular evolution theory of Section 3 yields the averaged evolution of the orbit up to a final time, e.g. 100 yrs after t_0. Through the solution of the variational equation for the averaged flow, and a formula similar to (1), we propagate the uncertainty of $\tilde{d}_{min}$ at the intermediate times (see Fig. 4(b)). Then we compute two curves interpolating the extrema of the possible distances at the intermediate steps (x and $*$ in the figure). The intersections t_1, t_2 of these curves with the t-axis define J.

The first method deals with non-linearity effects of orbit determination at time t_0, the second one takes into account the non-linearity effects of the orbit propagation. Hence these algorithms are complementary.

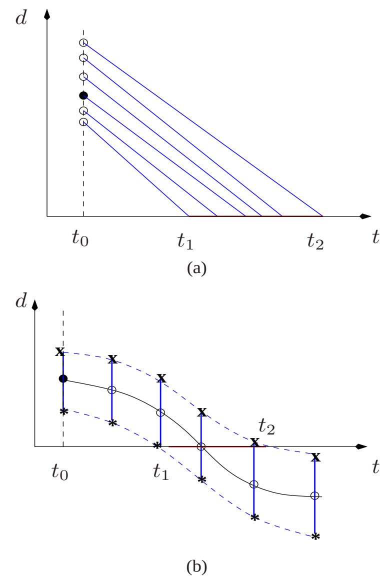

Figure 4: Computation of J (red line) with the first (top) and the second method (bottom).

6. Conclusions

Here we propose two algorithms useful to decide priority in the impact monitoring procedure, that deal also with non-linearity effects. These are based on recent analytical tools using the averaging principle. Reliability problems may arise in case of mean motion resonances or close encounters with a planet. We plan to perform a full test for these techniques.

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