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The librations of a triaxial, synchronously rotating planetary satellite

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Abstract

We compute the librations of Europa and Titan, extending Smith and Wahr's model for a biaxial, rotating Earth to a triaxial, synchronously rotating planetary satellite.

Introduction

The dynamical (i.e. orbital, rotational, and tidal) response of a planetary body to the gravitational pull of neighbouring celestial bodies reflects the structure of its interior. Evidence for putative oceans in the interior of Europa or Titan might therefore be gleaned from observations of their rotational behaviour.

As a result of tidal dissipation, many planetary bodies end up as satellites orbiting a parent body along a nearly circular orbit, locked in a state of synchronous rotation, and taking the shape of a triaxial ellipsoid. Librations (i.e. slight variations of the rotation rate) are the most striking irregularity displayed by the rotation of such planetary satellites. Because the triaxial shape favours the action of gravitational torques, it plays a significant role in the occurrence of librations and must be taken into account in rotation models.

The rotation of planetary bodies is usually modelled in one of the following two formalisms:

- **parameterised rigid body mechanics**, where rotation is considered independently of deformation, which is parameterised through a set of transfer functions (known as Love numbers or compliances), leading to finite-dimensional ordinary differential equations (ODE);
- **continuum mechanics**, where rotation and deformation are concurrently considered, leading to partial differential equations (PDE).

Although triaxiality is readily taken into account in the equations of rotation of parameterised rigid body mechanics, in the parameterisation of deformation it is not, and Love numbers and compliances are usually computed assuming spherical symmetry and neglecting rotation. For a full consideration of triaxiality, we extend Smith and Wahr's spectral treatment of continuum mechanics ([1] and [2]) to synchronously rotating planetary satellites, modelling rotation perturbations as infinitesimal toroidal displacements of degree 1 from a state of equilibrium (i.e. the state of steady synchronous rotation with respect to an inertial reference frame).

Method

We study the dynamics of the planetary satellite close to a triaxial reference configuration hydrostatically pre-stressed by its synchronous rotation under the permanent gravitational pull of the parent body. Small deviations from this reference configuration are described by linearisation of the equations provided by the laws of continuum mechanics, the laws of Newtonian gravitation, and additional assumptions on the rheology of the interior, which we expand up to the first order in the (polar or equatorial) flattenings. Spectral decomposition into spheroidal and toroidal components turns this system of PDE into an infinite-dimensional system of ODE: $\mathsf{Y}' = \mathsf{A}\mathsf{Y}$.

In the spherically symmetric, non-rotating, elastic, isotropic (SNREI) model, this infinite system is made up of an infinite number of finite subsystems (radial Rad_0^0, and for any degree $\ell \geq 1$ and order m, spheroidal Sph_ℓ^m and toroidal Tor_ℓ^m) decoupled from each other, as can be seen from a selected 124-dimensional submatrix (corresponding to degrees $0 \leq \ell \leq 3$) of the infinite ODE matrix A sketched in Fig. 1. In the biaxial, rotating, elastic, isotropic model, this infinite system is made up of an infinite number of infinite subsystems (orderwise) decoupled from each other (Fig. 2). Here, in the triaxial, synchronously rotating, elastic, isotropic model, this infinite system is made up of four

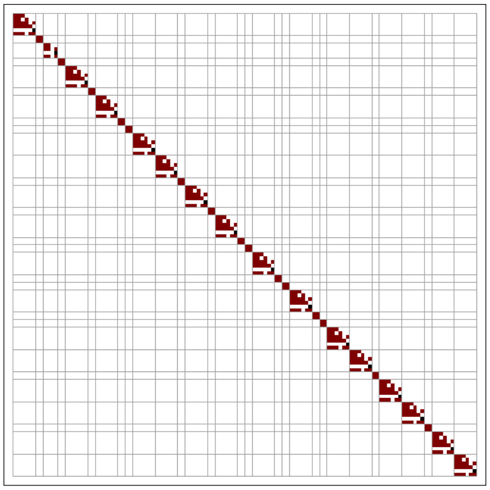

Figure 1: Spherical, non-rotating, elastic, isotropic (SNREI) A. The subsystems are sorted in the following order: Sph_2^{-2}, Tor_3^{-2}, Rad_0^0, Tor_1^0, Sph_2^0, Tor_3^0, Sph_2^2, Tor_3^2, Tor_2^{-2}, Sph_3^{-2}, Sph_1^0, Tor_2^0, Sph_3^0, Tor_2^2, Sph_3^2, Tor_3^{-3}, Tor_1^{-1}, Sph_2^{-1}, Tor_3^{-1}, Tor_1^1, Sph_2^1, Tor_3^1, Tor_3^3, Sph_3^{-3}, Sph_1^{-1}, Tor_2^{-1}, Sph_3^{-1}, Sph_1^1, Tor_2^1, Sph_3^1, Sph_3^3.

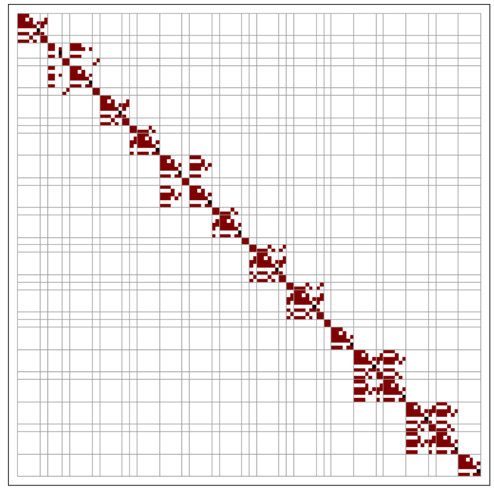

Figure 2: Biaxial, rotating A.

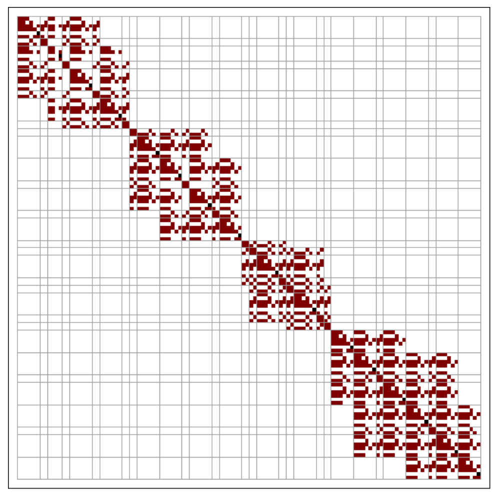

Figure 3: Triaxial, synchronously rotating A.

infinite subsystems decoupled from each other (Fig. 3). This extended coupling shows that rotation perturbations always naturally involve deformation.

Since we focus on librations, we assume the deformational component to be smaller than the rotational component (described by the subsystem Tor_1^0) by one order of magnitude in the flattenings. We may then truncate the infinite system to the first order in the flattenings, leaving the 30-dimensional subsystem $\mathsf{Rad}_0^0 \oplus \mathsf{Tor}_1^0 \oplus \mathsf{Sph}_2^{-2} \oplus \mathsf{Sph}_2^0 \oplus \mathsf{Sph}_2^2 \oplus \mathsf{Tor}_3^{-2} \oplus \mathsf{Tor}_3^0 \oplus \mathsf{Tor}_3^2$, suitable for numerical handling.

We compute the librations of Europa and Titan and compare our results with those produced by parameterised rigid body mechanics (e.g. [3] and [4]).

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