



# Mass estimates of long-period comets coming close to the Sun

Julio A. Fernández, Andrea Sosa

Departamento de Astronomía, Facultad de Ciencias, Iguá 4225, 11400 Montevideo, Uruguay (julio@fisica.edu.uy / Fax: +598 2525 0580)

## Abstract

We have estimated the masses of a set of Long-Period Comets (LPCs) (orbital periods  $P > 10^3$  yr) with perihelion distances  $q < 1.3$  AU and computed nongravitational parameters. We find masses in the range  $[0.5 \text{ } 10] \times 10^{12}$  kg, which correspond to diameters  $D$  in the range  $[1.4 \text{ } 3.6]$  km for an assumed bulk density of  $0.4 \text{ g cm}^{-3}$ . Even though large error bars may be attached to these estimates, for the time being we consider this as the first attempt to derive masses of a sample of LPCs in a consistent way. We have next studied the relationship between diameters and absolute total visual magnitudes  $H$ , and shown that it can be expressed by the lineal relation  $\log_{10} D(\text{km}) = 1.2 - 0.13H$ . In physical terms, this relation implies that the total comet brightness (nucleus + coma) goes as  $D^3$ . We finally discuss our results in broader terms, as regards the comet size distribution, and the minimum mass or size of an active comet.

## The method

Our method follows a procedure similar to that developed by Szutowicz and colleagues ([5], [6]) based on the computed nongravitational parameters,  $A_1$ ,  $A_2$ , from best-fit orbit solutions, instead of the change in the orbital period  $\Delta P$ , as done before for a sample of periodic comets ([3]).

Let  $\vec{J}$  be the nongravitational acceleration acting on a comet of mass  $M_N$ . If  $Q$  is the gas production rate,  $m$  the mass of the dominant sublimating molecule ( $\text{H}_2\text{O}$  in our case), and  $\vec{u}$  the effective outflow velocity, we have (considering modulus):

$$M_N = \frac{Qmu}{J} \quad (1)$$

where  $J = \sqrt{A_1^2 + A_2^2 + A_3^2} \times g(r)$  (it is usually assumed  $A_3 = 0$ ). The function  $g(r)$  describes the variation of the sublimation rate of water snow with the heliocentric distance  $r$  as adopted by [2].

The water production rate  $Q$  has been only determined for a few comets at a few points of their orbits. In order to have a better coverage of different comets, as well as different points along their orbits, we have determined a correlation between the observed  $Q$  values and the heliocentric total visual magnitudes  $m_h$  (see [4] for more details). Therefore, from the comet's lightcurve we can derive the water production rate  $Q(r_i)$  at any point of its orbit  $r_i$  in which the sublimation of water snow is significant ( $r \lesssim 2.5$  AU), and by means of eq.(1) we can compute  $M_N(r_i)$ . For each comet of our sample, we can compute the mean and the standard deviation of the 100 determinations along the orbit. From the computed  $\langle M_N \rangle$ , and by assuming a mean bulk density of  $0.4 \text{ g cm}^{-3}$ , we can derive the mean radius  $\langle R_N \rangle$  of the comet. We note that the standard deviation shown in Table 1 only considers the dispersion of the 100 computed  $M_N$  values along the comet's orbit, and not other factors of uncertainty as, for instance, in the assumed values of the physical parameters.

## Results

Table 1: Estimated mean radii and absolute total visual magnitudes

Comet	$\langle R_N \rangle$ (km)	H
C/1985 R1	$0.88 \pm 0.05$	8.3
C/1989 Q1	$1.00 \pm 0.12$	7.5
C/1993 Y1	$0.75 \pm 0.02$	8.7
C/1995 Y1	$1.14 \pm 0.02$	7.3
C/1999 J3	$0.71 \pm 0.01$	8.4
C/2002 V1	$1.57 \pm 0.16$	6.1
C/2004 Q2	$1.82 \pm 0.17$	5.2
C/2007 F1	$0.75 \pm 0.10$	8.0
C/2007 W1	$0.65 \pm 0.03$	8.5

The computed diameters are found to be well correlated to  $H$  through the linear relation

$$\log_{10} D(km) = 1.2 - 0.13H \quad (2)$$

By assuming that most of the comet brightness  $B$  was due to sunlight scattered by the dust particles in the coma, [1] found that it is related to the comet's diameter  $D$  through the relation

$$B \propto D^3 \quad (3)$$

when the limit of the coma is set by its fading into the sky background. By taking logarithms in eq.(3) and applying the definition of magnitude we arrive at an expression similar to our empirical derivation given by eq.(2).

From the lightcurves of LPCs, we were also able to compute the absolute total visual magnitudes, not only for the comets of Table 1, but for most of the LPCs with  $q < 1.3$  AU observed during the period 1970-2009. Most of the visual magnitudes were kindly provided by Daniel W. Green (see presentation by Sosa & Fernández for more details). We find that very few comets are fainter than absolute magnitude  $H \sim 13$ , which will correspond to a diameter  $D \sim 0.32$  km according to eq.(2). Our results confirm previous works (e.g. [7]) suggesting that a minimum size is required to have an active comet in the inner planetary region.

From the cumulative distribution of  $H$  found for our sample of LPCs, we can derive the cumulative distribution of comet diameters leading to a bimodal power law:

$$N(D) \propto D^{-s}, \quad (4)$$

where  $s \simeq 3.7$  for  $2.6 \lesssim D \lesssim 5.4$  km, and  $s \simeq 1.5$  for  $1.2 \lesssim D \lesssim 2.6$  km.

Very bright LPCs approaching or crossing Earth's orbit with negative values of  $H$  seem to be rare, at least over the studied period of 40 yr. Only one very bright comet appeared: C/1995 O1 (Hale-Bopp) with an estimated  $H = -1.7$ . By applying eq.(2) this corresponds to a diameter  $D = 26$  km which is slightly below some previous estimates (e.g. [8]), but still a sizable comet.

## References

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