

The Ill-Effects of Traditional Approximation in Exoplanet Atmospheric Flow Modeling

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Abstract

We investigate the effects of imposing the traditional approximation restriction in large-scale atmospheric flow models of exoplanets. In the traditional hydrostatic primitive equations, solved by general circulation and global climate models, terms associated with the horizontal components of the Coriolis acceleration and the metric terms are omitted. However, scale analysis suggests that the neglected terms may be up to $\sim 20\%$ of the included terms in the traditional equations.

In this work, we enhance a well-tested general circulation model to include the omitted terms consistently. In direct comparisons of simulations appropriate for hot extrasolar giant planets, we demonstrate a significant difference when the terms normally neglected in the approximation are retained; for example, large warm/cool regions (of up to $\sim 300\text{K}$) persist and large-scale jets disappear. The effect is most prominent when rotation does not strongly suppress vertical motion, as in most close-in exoplanets. It is also prominent during the early stages of any planet simulation, in which the atmosphere/climate is spun up from rest.

1. Background

Most general circulation models (GCMs) of extrasolar planet atmospheres are based on the standard primitive equations [1]. In addition to the hydrostatic balance, these equations make the *traditional approximation*, in which the “ $\cos \phi$ Coriolis” and “metric” terms from the full Navier-Stokes equations are concurrently omitted. For the Earth, the traditional approximation is valid to an acceptable degree—especially for adiabatic, balanced, synoptic-scale motions. However, the approximation is not as valid for other planets and for unbalanced flows. This can be particularly significant for exoplanets subject to strong diabatic forcing and/or unusual physical conditions (e.g., short-period

eccentricity, large radius, etc.). It is also significant, in principle, for simulations of any planetary atmospheres “spun up” from rest. In this work, we use a three-dimensional, stably-stratified hydrodynamics model that self-consistently retains the terms omitted in the traditional approximation to investigate the effects of relaxing the approximation in exoplanet simulations.

We use the equations of [2]. Their “non-traditional” primitive equations do not make the traditional approximation but still retain energy, axial angular momentum, and potential vorticity conservation (as in the traditional primitive equations). Here we discuss simulations of three tidally-synchronized model hot giant planets with different rotation rates: except for the rotation rate, the physical parameters of the planet are all taken to be those for HD209458b (see, e.g., [3]). Net heating due to the host star is represented with simple Newtonian relaxation, characterized by the relaxation time and the equilibrium potential temperature. Identical sets of simulations are performed using the “non-traditional” and “traditional” equations (hereafter WBE and HPE, respectively). Integration is carried out in all simulations from rest up to 100 planetary rotations. The vertical domain is from 1 to 1000 mbar, resolved by up to 20 layers in the vertical direction and T85 spectral resolution in the horizontal direction.

2. Results

Scaling analysis shows that rotation rate Ω strongly affects the importance of non-traditional effects, as do the characteristic scale height H and characteristic flow speed U (see, e.g., [2]). Specifically, the importance of keeping the $\cos \phi$ Coriolis terms can be assessed by considering the following comparison: $|2\Omega w \cos \phi|/|Du/Dt| \lesssim 2\Omega H \cos \phi/U$, where w is the vertical velocity and ϕ is the latitude. If this condition is true, $\cos \phi$ —as well as the metric—terms should be included. For the Earth, the differences be-

tween the WBE and HPE are small, except when diabatically forced, synoptic-scale motions in the tropics and planetary-scale motions are dominant. For the typical hot extrasolar giant planet (with HD209458b parameters), the difference is also minor, when $U \approx 1000 \text{ m s}^{-1}$. However, when $U \approx 100 \text{ m s}^{-1}$, the differences are not minor ($\lesssim 0.2$). For giant planets on shorter periods than HD209458b, the difference can be very significant (≈ 0.7). In fact, given the sensitivity to the initial condition observed in climate simulations starting from the rest state (see, e.g., [3]), the differences in all “spin-up” simulations of any planet is likely to be significant.

Figure 1 shows the difference between temperature and flow field of the direct simulation of HD209458b with and without the traditionally neglected terms. In the simulations, the location of vortices and hot regions for WBE is different than for HPE. This is because relaxing the traditional approximation introduces, in addition to a rotationally-modified Rossby mode, smaller scale perturbations. The small-scale fluctuations in the flow obtained with WBE is generated by the vertical velocity terms absent in HPE.

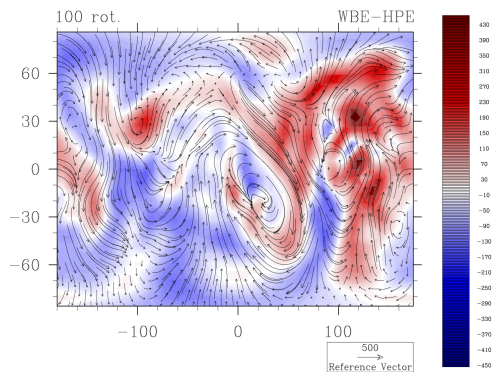


Figure 1: Cylindrical-equidistant projection of the difference between the WBE and HPE fields at the 475 mbar level. The temperature and flow fields at 100 planetary rotations are shown superimposed.

The basic comparison presented above is repeated with varying Rossby number Ro , which measures the ratio of U to Ω for a fixed characteristic length scale (planet radius here). For large Rossby number ($Ro \approx 1.5$), the differences in the flow structure are even more noticeable than for the moderate Rossby number ($Ro \approx 0.5$) case, shown in Figure 1. For small Rossby number ($Ro \approx 0.1$) case, the differences in the flow structure are less than in the moderate and large

Ro cases. The main difference in the small Ro case is that the WBE simulation does not exhibit a strong westward polar jet, as in the HPE simulation. Both WBE and HPE simulations show a more zonal flow, compared with the moderate and large Ro cases—as expected for small Ro flows in general.

3. Conclusion

The traditional approximation in general circulation models of hot exoplanets leads to results which are significantly different, compared with results obtained with models not making the approximation. This presents another cautionary tale for judicious application of models, theoretical and numerical, originally developed for Solar System planets (Earth in particular): equations with full Coriolis force and metric terms should be used to obtain more robust results for exoplanets. We have used HD209458b as a paradigm hot extrasolar planet to investigate the significance of neglecting certain terms in the hydrodynamics equations. However, the consequences can also be significant for any planet which possesses flows with fast vertical and/or slow horizontal motions. Future work will investigate these effects more carefully when the small scales are better represented with higher resolution.

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