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The flow of granular materials versus the angle of repose

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Abstract

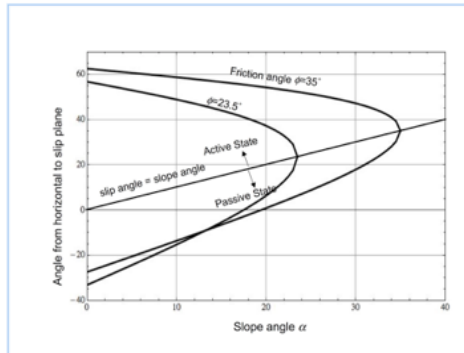
A common concept from the continuum theory of granular materials is that of the angle of repose: the maximum angle a pile of material will achieve when poured slowly onto a flat surface. It is easily proved to be equal to the angle of internal friction ϕ defined in the common Mohr-Coulomb (MC) model. Some assume that flow is not possible at any lessor angle; so they conclude that a material found to flow to small slope angles cannot be modeled with the continuum theory without special assumptions relating to lubrication or fluidization. Here a detailed analysis of that concept is presented using a combination of theory and a numerical simulation of the granular cliff collapse problem as a detailed example. It is found that flow can occur at many angles both less than (in the static case) and greater than (in the dynamic case) the angle of repose. Thus, in the cliff collapse problem, for a material with an angle of friction of 35˚, flow does continue to final slope angles as small as 10˚.

1. Introduction

So can flows occur in the MC theory with surface slope angles θ less than the angle of repose? How can the cliff collapse problem produce final slopes much smaller than the angle of repose? These are the topics analysed here, both for a static case and then for the highly dynamic collapse of a granular cliff.

2. Failure Of Static Surface Slopes

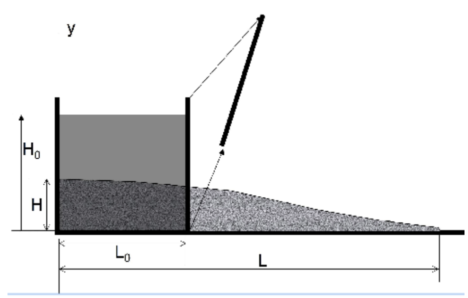
A simple Mohr's circle analysis provides valuable information about possible failure states according to the Mohr-Coulomb theory. Specifically, it can be used to show that failure depends not only on the angle of the slope, but also on the normal stress on a plane perpendicular to the slope (the "down-slope" stress). It is the variety of values for that down-slope stress that provides the variety of failure slopes. Only one special value for the down-slope stress sets the flow for the angle of repose example.



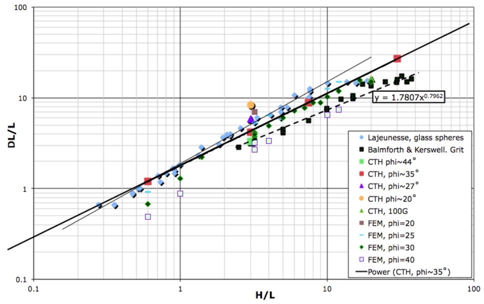
In the general case, failure can occur with a free surface at any angle $\alpha \leq \phi$ and the failure will be along another slope θ above or below that of the free surface. The plot shows the relation between the slope angle and the slip angle for two specific values of the angle of internal friction.

3. The Cliff Collapse Problem

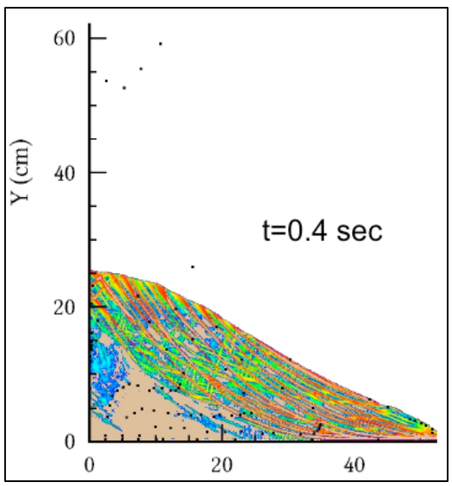
An exemplary example that illustrates these concepts is that of the collapse of a vertical surface (a cliff) of a granular material, for which experiments and code calculations are available in the literature.



The results of a numerical simulation by the author are presented in the next figure, using a friction angle of 35˚. At time t=0, a right wall is removed and the granular material collapses and flows downward and to the right. This next plot shows the final length of the run-out, which it is in excellent agreement with the experimental data shown.

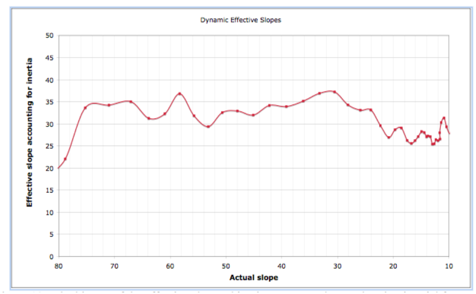


An important detail of the process during the cliff collapse is presented in the next figure. It shows the contours of the plastic strain rate at a particular time during the process. Note how the dominant failures (red bands) are along planes steeper than the slope at the upper steeper slope locations and less than the



surface slope near the more shallow surface slopes, creating a "circular arc" failure surface, as is often assumed *a-priori* for the prediction of landslides. That shows that the simulations exhibit distinctive strain localization ("shear banding") and slide failures along planes predictable form the first figure.

Finally, the last figure illustrates the important role of the inertia in this dynamic problem. Recall that in the equations of motion, the gravity and the inertial forces always combine. Thus, it is the sum of those two forces against which the actual slope angle should be measured, it gives the direction of "instantaneous" down" at any location. That determines the "effective" slope angle, not the geometric slope angle relative to gravity only. In the numerical simulations, the local "down" direction has been determined as a function of time at one single particle that follows along the free surface during the collapse. It begins at a free surface with an initial geometric slope beginning at 90˚, and during the collapse ranges to a final slope of only about 10˚. However, after taking the inertial forces into account, the figure shows that the actual "effective" slope is always less than the angle of friction, just as the theory requires. This shows the importance of the dynamics in this collapse problem.



Acknowledgements

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