EPSC Abstracts
Vol. 6, EPSC-DPS2011-171, 2011
EPSC-DPS Joint Meeting 2011

Dynamic Elastic Tides

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Abstract

We present a formulation for solid body tides that includes the frequency dependence of the tidal dissipation in a self-consistent, modular way. We derive general expressions for tidal heating and rate of change of semimajor axis and eccentricity. We then specify a Kelvin-Voigt rheology, which corresponds to a constant time lag tidal model commonly used in the literature. We verify that our model produces the classical expressions and present several new expansions to high eccentricity.

Description

Traditionally, solar system studies of tides have relied on the constant time lag model, which is the only tidal model with published analytic (not numerical) expressions for tidal heating and orbital decay at high eccentricity [8]. In this model, tides are raised by an imaginary perturber displaced by a constant time lag along the orbit from the actual perturber. If the orbit expands due to the effect of tides, the time lag is constant and so the phase lag is forced to decrease along with the orbital frequency.

The constant time lag model predicts that tidal dissipation is linearly proportional to the orbital frequency for small phase lags. If we define the tidal quality factor $Q = 1/\tan\delta$ where δ is the phase lag, then the constant time lag model predicts that Q is proportional to $1/n$ where n is the orbital frequency. Unfortunately, this frequency-dependence is not in agreement with the few measured values.

For instance, the measured Q of the Moon is roughly frequency-independent over the limited frequency range studied. Williams [7] reports a Q of approximately 30 for a forcing frequency of one month and a Q of about 35 for one year. Terrestrial geophysicists have studied the Earth over a broader frequency range and have found more complicated frequency dependences, such as described by the Andrade model or Burgers model. Planetary scientists do not understand the frequency dependence of Q, but as far as we do, the constant time lag model seems like a poor choice. The widespread use of the constant time lag model is due to its mathematical tractability, not to any physical evidence in favor of it.

Models of stellar tides take a different approach. Stellar tides are generally described as the excitation of various modes of the star. The theory of the equilibrium tide was pioneered by Zahn (e.g. [9]), who computed the viscous dissipation resulting from the velocity field in turbulent convective zones in the star. If dissipation is dominated by the equilibrium tide, the fundamental and acoustic modes of the star are the modes excited by the lower frequency tidal forcing. All dissipation occurs in turbulent regions.

The theory of the dynamical tide describes the response of the star to modes of higher frequency than the fundamental modes. For example, Terquem et al. [6] and Barker and Ogilvie [1] examine the excitation of g-modes (gravity modes) and resonances between the tidal forcing and the normal modes of the star. g-modes have a restoring force due to buoyancy. Inertial modes have a restoring force from the Coriolis force and are important contributors to stellar dissipation [4, 3]. Recently, Penev and Sasselov [5] have re-examined the equilibrium tide and constrained the tidal quality factor that applies for extrasolar planets.

We describe a new formulation of solid body tides that models tidal displacements as a sum of excited elastic modes, analogous to the modeling of stellar tides as excited vibrational modes. The assumptions about the tidal frequency dependence enter near the end of the calculation in a modular and mathematically clean manner. This modularity will allow us to easily compare tidal dissipation and its effects for different rheologies in future work. Here we describe the theory and derive general expressions for wobble damping, tidal heating, tidal despinning, and rate of change of semimajor axis and eccentricity for a system with a zero-obliquity perturber in an eccentric, noninclined orbit. We then specify a Kelvin-Voigt rheology, which corresponds to the constant time lag model, and verify our model with the classic results.

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