

Marginal stability of planetary convection

S.V. Starchenko

IZMIRAN, Troitsk Moscow region, 142190 Russia (sstarchenko@mail.ru / Fax: +7-496-7510124)

Abstract

Review of the known and recent researches is presented for the onset of Boussinesq convection in rapidly rotating spherical shell imitating the planetary interiors. Those researches and the first principles of well-mixed compositional and/or almost adiabatic thermal *non-Boussinesq* convection in the planets allow deriving new results for marginal convection stability and planetary magnetic field excitation.

1. Review

The study of Boussinesq rotating convection in self-gravitating spheres and shells at a small Ekman number E has developed a long way since the pioneering studies of Roberts [5] and Busse [1]. Their key conclusion was that the onset of convection is localized at mid-latitudes in cells arranged on a cylinder whose generator is parallel with the rotation axis. This 'cartridge belt' therefore consists of axial rolls whose structure varies slowly between the equatorial plane and the intersection of the cylinder with the outer boundary. The rolls have a short $E^{1/3}$ azimuthal length scale and propagate prograde. However the detail of the radial structure was not straightforward and its correct form was only resolved much later by Jones, Soward & Mussa [4]. Their asymptotic solution describes rolls that extend radially in the direction normal to the cylinder on a relatively long $E^{1/6}$ length scale; moreover these elongated structures also tilt progradely with increasing distance from the axis. The case of differential heating, when there are no heat sources in the fluid as in [1, 4 and 5] but instead temperature difference is maintained between the inner and outer boundaries, was considered in [3, 2, 7 and 8]. Then relatively large temperature gradients occur close to the inner boundary. As a result the cartridge belt now resides on the cylinder tangent to the inner boundary at its equator. The rolls now have the radial length scale $E^{2/9}$ and tilt as before. Certainly this general

picture is now widely accepted. However the Boussinesq approximation itself is inadequate to model well-mixed compositional [6] and almost adiabatic [7, 8] thermal convection in the planets and the anelastic approximation is often better especially for the giant planets. Nevertheless, much of the basic dynamics can be investigated by proper extensions of the simple thermal Boussinesq models [3, 2, 7 and 8].

2. New results

The energy estimates for well-mixed compositional convection from [6] and large Prandtl number ($Pr \gg 1$) solutions from [7, 8] allow me roughly to model Rayleigh Ra and critical Ra_{cr} Rayleigh numbers for the Terrestrial planets. This result is shown in Figure.

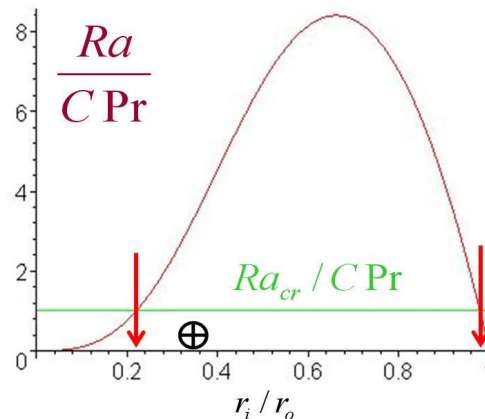


Figure: For compositional convection in the Terrestrial planets modified Rayleigh numbers Ra and Ra_{cr} are shown in dependence on their inner to outer core radius ratio r_i/r_o , Prandtl Pr number and large constant C . The modern Earth with vigorous compositional convection is indicated. Venus could be on the left or near the left red arrow, while Mercury and Mars are perhaps near the right arrow.

It is clear from Figure that compositional convection could exist only when the inner rigid core has grown enough to overcome dissipative effects. So the radius of the Venus inner core could be lower or just about the radius shown by the left red arrow in Figure. This naturally resolve the problem of absence of the own magnetic field in Venus. Besides the rather low supercritical level for the Earth's type compositional convection (see $Ra/Ra_{cr} \approx 3$ in Figure) explains the success of the recent numerical geodynamo modeling. However the too small inner core age problem is strongly amplified if the inner core is crystallized from the liquid outer core as it is widely accepted (alternative view resolving this problem in Pushkarev and Starchenko report on this TP 12 section). The planets with thin enough liquid core also could have no compositional convection (see the right arrow in Figure), while the transition here is very sharp. This could result in the absence of own modern magnetic field as in Mars, Moon and the majority of large satellites of the giant planets. As well due to this sharp transition small and unusual magnetic field of Mercury and Ganymede could be produced by compositional convection in cooperation with tidal effects perhaps. New results for critical frequencies and convection dynamics also will be presented by me for this case of pure compositional convection.

The well-mixed compositional and/or almost adiabatic planetary convection problem could be resolved by the PDE equations those are formally similar to the Boussinesq PDEs, but the boundary conditions remain different. However for the marginal stability this difference is not crucial. Thus I am going to show how it is possible to use for different planets the large numerical and analytic experience collected for the Boussinesq models.

Acknowledgements

This work was supported by RFBR grant No. 09-05-00979-a.

References

- [1] Busse, F.H.: Thermal instabilities in rapidly rotating system, *J. Fluid Mech.*, Vol. 44, pp. 441–460, 1970.
- [2] Bassom A.P., Soward A.M., Starchenko S.V.: The onset of strongly localized thermal convection in rotating spherical shells, *J. Fluid Mech.*, submitted in 2010.
- [3] Dormy, E., Soward, A.M., Jones, C.A., Jault, D. and Cardin, P.: The onset of thermal convection in rotating spherical shells, *J. Fluid Mech.*, Vol. 501, pp. 43–70, 2004.
- [4] Jones, C.A., Soward, A.M. and Mussa, A.I.: The onset of thermal convection in a rapidly rotating sphere, *J. Fluid Mech.*, Vol. 405, pp. 157–179, 2000.
- [5] Roberts, P.H.: On the thermal instability of a highly rotating fluid sphere, *Phil. Trans. R. Soc. Lond. A*, Vol. 263, pp. 93–117, 1968.
- [6] Starchenko S.V.: Gravitational differentiation in the planetary cores, *Russian Journal of Earth Sciences*, Vol. 5, No 6, pp. 431–438, 2003.
- [7] Starchenko S.V., Kotelnikova M.S.: Marginal stability of convection in rapidly rotating and thick spherical shell, *Zh. Eksp. Teor. Fiz.*, submitted in 2011.
- [8] Starchenko S.V., Kotelnikova M.S., Maslov I.V.: Marginal stability of almost adiabatic planetary convection, *Geophys. Astrophys. Fluid Dynamics*, Vol. 100, pp. 397–428, 2006.