Optimal asteroid mass determination from planetary range observations; a study of a simplified test model.

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Abstract

Mars ranging observations are available over the past 10 years with an accuracy of a few meters. Such precise measurements of the Earth-Mars distance provide valuable constraints on the masses of the asteroids perturbing both planets. Today more than 30 asteroid masses have thus been estimated from planetary ranging data (see [1] and [2]). Obtaining unbiased mass estimations is nevertheless difficult. Various systematic errors can be introduced by an imperfect reduction of spacecraft tracking observations to planetary ranging data. The large number of asteroids and the limited a priori knowledge of their masses is also an obstacle for parameter selection. Fitting in a model a mass of a negligible perturber, or on the contrary omitting a significant perturber, will induce important bias in the determined asteroid masses. In this communication, we investigate a simplified version of the mass determination problem. Instead of planetary ranging observations from spacecraft or radar data, we consider synthetic ranging observations generated with the INPOP [2] ephemeris for a test model containing ~25000 asteroids. We then suggest a method for optimal parameter selection and estimation in this simplified framework.

The test model

The test model of the Solar System is based on the dynamical model of INPOP and contains 27142 asteroids. The selection corresponds to all the known objects with diameters larger than approximately 10 km. For each asteroid, we use available astrophysical data to estimate a mass and a corresponding uncertainty. The estimations rely on an assumed density of 2.5 g·cm^{-3}. For asteroids with radiometric determinations of diameters, the estimated masses may thus differ from the correct values by a factor of 2. With only a loose constraint on albedo provided by taxon-omy, the uncertainty factor grows to 5. In the absence of any particular information, the uncertainty factor is estimated at 16. Integrating the test model between 1960 and 2010, we are able to generate synthetic observations of planetary ranges. The observations are restricted to planets and time intervals that correspond to actual data. For example we use only the 1976-1983 and 1999-2010 time spans for Mars ranging. The observational error is assumed Gaussian with amplitude based on standard deviation of residuals in [2].

Problem statement

Our objective in this study is to determine the constraints imposed by the synthetic ranging observations on the asteroid masses in the test model.

Linear formulation

We verify that the perturbation of the Earth-Mars distance induced by mutually interacting asteroids is equal in practice to the sum of the individually induced perturbations. Thus the dependency on asteroid masses of the Earth-Mars distance can be considered linear. We can reasonably assume that this linearity applies for any Earth-planet distance. With n the total number of asteroids in the test model, $\beta = (\beta_1, ..., \beta_n)$ the asteroid masses, and z the corresponding synthetic ranging observations, we can write

$$z - z_0 = M\beta. \tag{1}$$

$M = (M_1, ..., M_n)$ is the matrix of the partial derivatives with respect to asteroid masses and z_0 are ranging observations obtained with $\beta = (0, ..., 0)$. A large part of asteroid perturbations can be absorbed into small changes of planetary initial conditions. The effect is accounted for by determining the vector space spanned by the partials with respect to planetary initial conditions and by projecting on the orthogonal complement of this vector space, both $z - z_0$ and the columns

of M. We show that the distribution of an asteroid mass around its value obtained from astrophysical data follows approximately a truncated Gaussian distribution. By assuming an untruncated distribution, some of the a priori information is left out, but the solution of our problem can be obtained analytically. The constraints imposed by the ranging data are then represented by new, updated Gaussian distributions of asteroid masses. The mean values β_{po} and the variances σ_{po}^2 of these a posteriori distributions are given by the solution of Tikhonov regularization [3]. With $\Delta z = z - z_0$, we have

$$\beta_{po} = (M^T M + C^{-1})^{-1}(M^T \Delta z + C^{-1}\beta_{pr}), \quad (2)$$

where β_{pr} and C represent respectively the mean values and the (diagonal) covariance matrix of the a priori distributions. The a posteriori variances are given by

$$\sigma_{po}^2 = \text{diag}\left[(M^T M + C^{-1})^{-1}\right]. \quad (3)$$

In both equations, the standard deviation of the observational error is taken equal to 1. This is achieved by properly weighting the observations.

Parameter selection

All the 27142 asteroids have to be considered in the above equations, otherwise systematic error is introduced. In practice, the number of asteroids in the Solar System is virtually infinite. Thus when dealing with real data, parameter selection is necessary and some amount of systematic error is unavoidable. In [4] we have shown that a selection of less than 300 individual asteroids and a solid ring are able to represent the total asteroid perturbation induced on ranging observations down to a few meters. In the test model, the selection process corresponds formally to the search of $n + 1$ constrained parameters $\alpha = (\alpha_1, ..., \alpha_n)$ and α_{n+1} that minimize

$$|M(\alpha - \beta) + \alpha_{n+1}M_r|^2. \quad (4)$$

M_r represents the partial derivative of the ranging observations with respect to the mass of a circular ring. The parameter α_{n+1} corresponds to the mass of the ring and is constrained positive, all other α_i can be equal to zero or to β_i. Selection is achieved by limiting the number of non zero α_i to some reasonable value N. In the following, we choose $N = 300$. The constrained minimization of (4) is solved with appropriate software in a branch and bound approach as in [4]. We thus obtain a list of less than 300 asteroid masses that may be determined individually with

a minimum amount of systematic error. By performing the selection for varying random asteroid masses in the test model, we estimate for each asteroid a probability of being selected for individual mass estimation. We use this probability to modify the a priori asteroid mass distributions. Except for a list of approximately 300 asteroids, the modified a priori distributions are very close to zero centred Dirac distributions. These asteroids can be safely ignored in the mass determination process.

Results

Constraints imposed by ranging observations on the previously determined list of masses can be estimated with equation (3). But this approach ignores the small systematic error necessarily introduced by parameter selection. We obtain a more realistic estimate based on randomly generated sets of reasonable asteroid masses. For the asteroids in the determined list, the random masses are compared with the estimates provided by equation (2). Repeating this process several times for different sets of random masses, we show that 23 masses are on average determined within 30% of the correct values. Up to 129 masses are determined on average to better than 50%. These uncertainties correspond to the maximum constraints on asteroid masses that can expected from non-synthetic ranging observations.

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