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YORP on tumbling asteroids

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Abstract

A semi-analytical model of the Yarkovsky-O'Keefe-Radzievskii-Paddack (YORP) effect on an asteroid spin in non principal axis rotation state is presented. Assuming zero conductivity, the YORP torque is represented by spherical harmonics series with vector coefficients, allowing to use any degree and order of approximation. Within the quadrupole approximation of the illumination function we find the same first integrals involving rotational momentum, obliquity and dynamical inertia that were obtained by Cicaló and Scheeres [1]. The integrals do not exist when higher degree terms of illumination function are included and then the asymptotic states known from Vokrouhlický et al. [2] appear. This resolves an apparent contradiction between earlier results. Averaged equations of motion admit stable and unstable limit cycle solutions that were not detected previously.

1. Introduction

The YORP effect describes the influence of the net radiative torque on the rotation of a small body like an asteroid or a meteoroid [3]. Apart from two exceptions, it has always been studied under the assumption of rotation around the axis of maximum inertia. The first hint that this assumption is unreliable was given in the numerical experiments of Vokrouhlický et al. [2]; allowing a general rotation perturbed by the YORP, they found that in most of cases the angular momentum vector tends to an asymptotic state of precession in the body frame, combined with a constant angle with respect to the orbital plane and an infinite increase of angular rate. Cicaló and Scheeres [1] developed an analytical model of the same problem, but their results contradicted [2]: the asymptotic states manifested themselves as equilibria in a generally stationary solution not revealing any systematic trend. Our present work offers a more complete analytical solution of the YORP effect explaining the difference between [2] and [1].

2. YORP torque model

In the absence of conductivity, the only serious problem related with modelling the YORP torque for the purpose of analytical treatment is related with handling a non-smooth illumination function, containing $\max(0, \hat{\mathbf{n}} \cdot \hat{\mathbf{r}}_{\mathrm{s}})$, where $\hat{\mathbf{n}}$ is the unit surface normal vector, and $\hat{\mathbf{r}}_{\mathrm{s}}$ is the unit vector towards the Sun. Cicaló and Scheeres [1] approximated it as a quadratic polynomial of $\hat{\mathbf{n}} \cdot \hat{\mathbf{r}}_{\mathrm{s}}$. In our study, an arbitrary degree/order spherical harmonics $(Y_{n,m})$ series were used, so that the torque $\mathbf{M}$ is given by

$$\mathbf{M} = -K \sum_{n \geq 1} \sum_{m=-n}^{n} \mathbf{v}_{n,m} Y_{n,m}(\hat{\mathbf{r}}_{\mathrm{s}}), \qquad (1)$$

where K is a coefficient incorporating physical constants and distance from the Sun, and vectors $\mathbf{v}_{n,m}$ can be precomputed for a given object by means of integrals over its surface

$$\mathbf{v}_{n,m} = \frac{4\pi}{2n+1} \frac{c_n}{V} \oint Y^*_{n,m}(\hat{\mathbf{n}})\, \mathbf{r} \times \hat{\mathbf{n}}\, \mathrm{d}S, \qquad (2)$$

where V is the volume, $c_1 = 1/2$, all remaining $c_{2n+1} = 0$, and

$$c_{2n} = -\frac{(4n+1)\, P_{2n}(0)}{(4n+4)\,(2n-1)}. \qquad (3)$$

3. Averaged equations of rotation

The departure system of equations

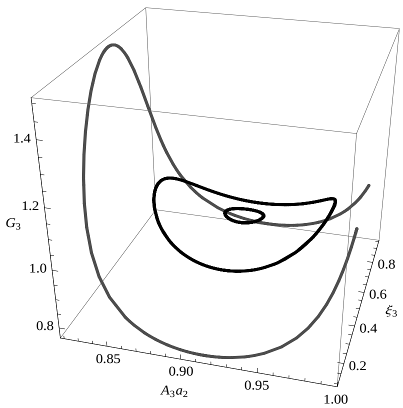
$$\dot{\Delta} = \frac{2\Delta}{G} \left(\hat{\mathbf{t}}_3 \cdot \mathbf{M} - \frac{\Delta}{G} \boldsymbol{\Omega} \cdot \mathbf{M} \right), \qquad (4)$$

$$\dot{G} = \hat{\mathbf{t}}_3 \cdot \mathbf{M}, \qquad (5)$$

$$\dot{\varepsilon} = -\frac{\hat{\mathbf{t}}_2 \cdot \mathbf{M}}{G}, \qquad (6)$$

concerns total angular momentum $G = ||I\boldsymbol{\Omega}||$, obliquity ε and 'dynamical inertia' $\Delta = G^2/(\boldsymbol{\Omega} \cdot I\,\boldsymbol{\Omega})$, where I is the matrix of inertia and $\boldsymbol{\Omega}$ is the angular velocity vector. Unit vector $\hat{\mathbf{t}}_3 = \mathbf{G}/G$, and $\hat{\mathbf{t}}_2$ lies in the plane normal to $\mathbf{G}$, $90°$ from the ascending node of this plane on the orbit.

Averaging with respect to Keplerian orbital motion, precession/nutation, and intrinsic rotation we have obtained two sets of secular equations: one in the Short Axis Mode (SAM – circulation of $\mathbf{G}$ around the maximum inertia axis) and one in the Long Axis Mode (LAM – circulation around the minimum inertia axis). When truncated on the second degree of spherical functions, our expressions are identical with the solution from [1]. But this truncation admits symmetries that do not exist in higher degrees which explains the surprising behavior of the Cicaló-Scheeres model and confirms one of its possible explanations formulated in [1].



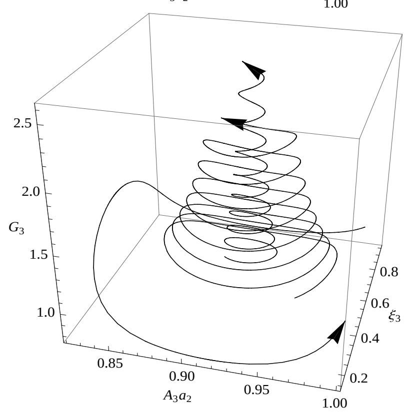


Figure 1: Exemplary secular solutions in the space of G, $A = \Delta^{-1}$, and $\xi = \cos\varepsilon$. Top: Quadratic truncation, bottom: harmonics up to degree 10. Subscript 3 refers to the SAM rotation. Factor a_2 is the intermediate moment of inertia ($Aa_2 = 1$ is the SAM/LAM separatrix value).

4. Results and Conclusions

Figure 1 illustrates the difference between quadratic truncation and a general case of the present solution. On one hand, we confirm the scenarios from numerical experiments of [2]: asymptotic states with obliquities close to $55°$ or $125°$ and related values of Δ are indeed attractors for a large number of initial conditions, especially if they are chosen close to the principal axis rotation, like it was done in [2]. On the other hand, we have found stable and unstable limit cycles surrounding the asymptotic states on the plane of obliquity and dynamical inertia. They passed undetected by Vokrouhlický et al. due to selective choice of initial Δ. Stable limit cycles offer another possibility of an asymptotic state, but similarly to the earlier findings of [2], they involve an unlimited growth of G. Any state with systematically decreasing G is transient and exits its SAM or LAM zone for a sufficiently low angular momentum.

Although our analytical model resolves the controversy between [2] and [1], its conclusions are still troublesome for the understanding of small asteroids rotation statistics. An inevitable onset of tumbling and unlimited speed-up of rotation contradict the observations: the vast majority of asteroids rotate close to the principal axis mode. It means that dissipative phenomena should be considered together with YORP in order to match the observed dynamics.

References

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