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Ground-track resonances around Vesta: potential threats for Dawn?

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Abstract

The aim of Dawn mission is the acquisition of data from orbits around (4)-Vesta and (1)-Ceres, the two most massive asteroids. Due to the low thrust propulsion, Dawn will slowly transit through ground-track resonances (abbreviated by *GTR*), where the perturbations on Dawn orbit may be significant. In this context, to safety go the Dawn mission from the approach to the lowest orbit, it is essential to know the properties of the crossed resonances. We analytically investigate one of the properties of the 2:3 *GTR*. This resonance makes the eccentricity of Dawn increase until 0.2. Then the radius of the Dawn orbit changes about 260 km dangerously leading Dawn nearer to Vesta. We determine that this increases is due to the C_{22} coefficients (characterizing the equatorial ellipticity of the asteroid).

1. Introduction

Dawn is a mission to explore main belt asteroids in order to yield insights into important questions about the formation and evolution of the solar system [3]. The dynamics of a probe orbiting within $\sim$ 1 000 km from Vesta is dominated by the gravitational field of Vesta [4]. Due to the low thrust propulsion, Dawn will slowly cross and transit through *GTR* due to the non uniformity of the gravity field of Vesta. In these resonances the perturbations on Dawn orbit may be significant [1, 4]. In particular in the 2:3 resonance the eccentricity increase up to 0.2 [1].

2 Numerical analysis

In Fig. 1, we plot the results of the numerical integrations over a time span of 1 year. The fixed initial conditions are $i = 90°$, $\Omega = \omega = 0°$ and $\theta = 0°$ for the sidereal time. These figures allow to locate the *GTR*. Indeed, when the orbital period of the space probe is close to a commensurability with the rotational period

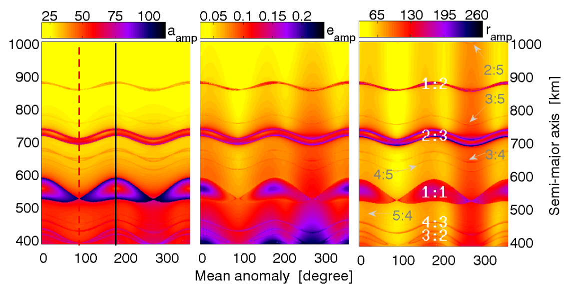

Figure 1: The amplitude of variation (colorbars) of the semi-major axis, of the eccentricity and of the radial distance computed as functions of the initial mean anomaly and the initial semi-major axis. The equations of motion include the central body attraction, the harmonics until degree and order 4.

of Vesta, the ground-track of the probe will periodically encounter the same configuration of the gravitational field. This is a gravitational or ground-track resonance (*GTR*). Then a part of the effects of the gravitational perturbations is amplified leading to a motion with very long period and large perturbations in the semi-major axis of the spacecraft orbit. This is what we observe in the left panel of Fig. 1 where the variation of the semi-major axis is clearly the largest, at 550 km and 720 km. The results obtained here are similar to [4]. We find 5 main *GTR*. In the right panel of Fig. 1, we notice that the largest resonance is the 1:1 but the strongest perturbations on the radial distance appear in the 2:3 resonance. We can deduce from the central panel that the high value of the amplitude of variation of the radial distance in the 2:3 resonance is due to an increase of the eccentricity.

3 Analytical study

The Hamiltonian describing the motion of a probe around an irregular central body is given by [2] using the eccentricity functions and the inclination functions. A *GTR* occurs when there is a commensurability between the orbital period of the probe and the rotational period of the asteroid. The properties of *GTR* around Vesta are already studied in [1]. The Hamiltonian of the 2:3 resonance, until the first order in eccentricity, is given by ($\sigma = 3(\Omega + \omega + M) - 2\theta$) [1]:

$$\mathcal{H}_{2:3} = -\frac{\mu^2}{2L^2} - \frac{15\mu^5 R_e^3}{8L^8}\Big(- S_{32}\cos(\sigma - \Omega) \\ C_{32}\sin(\sigma - \Omega)\Big) - e\frac{21\mu^4 R_e^2}{8L^6}\Big(C_{22}\cos(\sigma - \omega - \Omega) \\ + S_{22}\sin(\sigma - \omega - \Omega)\Big) + \dot{\theta}\Lambda\,,$$

where R_e is the equatorial radius of Vesta, $\dot{\theta}\Lambda$ is the asteroid rotation and C_{nm}, S_{nm} are the coefficients of the spherical harmonic gravity field representation [2]. With this Hamiltonian we can qualitatively give the secular equation of the eccentricity $\dot{e} \approx |\frac{\partial \mathcal{H}}{\partial \omega}|$. To have an idea of the variation of the eccentricity during the motion we can look at the $\dot{e}/e$ value:

$$\dot{e}/e \approx e^{-1}\,\partial H/\partial\omega \propto 21\mu^4 R_e^2/(8L^6)\,C_{22}\,. \qquad (1)$$

Then the variation of the eccentricity depends on one of the biggest coefficients of the gravity field C_{22}. For Vesta, this coefficient C_{22} is 5 times greater than the coefficient C_{32} responsible for the 2:3 *GTR*. We evaluate (linked to Vesta), near to the 2:3 *GTR*, the multiplying coefficient appearing in front of the two major contributions C_{32} and C_{22}:

$$\frac{15\mu^5 R_e^3}{8L^8}C_{32} \approx 6\,913 \quad \& \quad e\frac{21\mu^4 R_e^2}{8L^6}C_{22} \approx 25\,179\;e\,. \qquad (2)$$

In the case of Dawn orbiting Vesta, this highlights the importance of this additional term C_{22} in the 2:3 resonance.

Now we numerically verify our conclusion. In the left panel of Fig. 2, we only use the J_{32} ($J_{nm} = \sqrt{C_{nm}^2 + S_{nm}^2}$) term to modelize the central body. We see a weak increase of the eccentricity (0.03). In the central panel, we add the J_{22} term and we notice that the eccentricity reaches higher values(0.15). In the right panel, the gravity field contains also the $J_2 = -C_{20}$ term and the maximum eccentricity reached is 0.2. Then, with the help of Fig. 1 and Fig. 2, we can conclude that the major part of the increase of the eccentricity in the $2:3$ *GTR* is due to the J_{22} coefficient. Finally, we numerically simulate the slow descent, with a continuous -20mN thrust, of Dawn from a radius of 1 000 km to 600 km. We make this test with different gravity field configurations. The first crossed *GTR* is the 2:3. The eccentricity increases only in the full model and in the models including the coefficient.

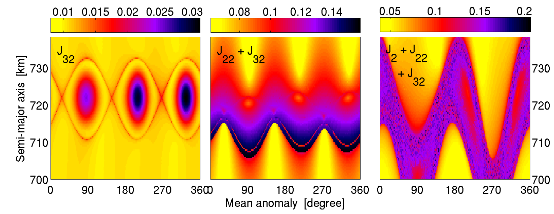

Figure 2: Around the 2:3 *GTR*. The amplitude of variation of the eccentricity (colorbars). The equations of motion include the harmonic J_{32} (left panel); the harmonics J_{22} and J_{32} (central panel); and the harmonics J_2, J_{22} and J_{32} (right panel).

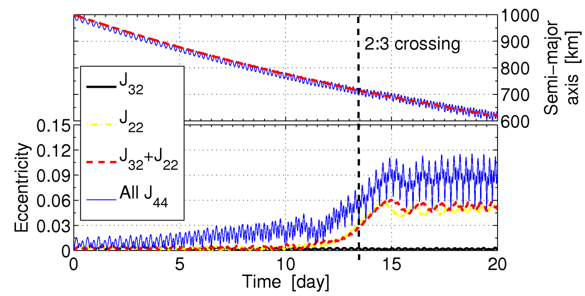

Figure 3: Evolution of the semi-major axis and the eccentricity of Dawn going down from 1 000 km to 600 km radius with a thrust of -20mN. The increase of the eccentricity depends on the gravity field model.

4. Summary and Conclusions

The *GTR* around Vesta has been numerically studied in [4, 1]. The properties (location, aperture, period) of these *GTR* has been analytically study in [1]. We show in this paper the threat of the $2:3$ *GTR* that can increase the eccentricity of Dawn (because of C_{22}) involving a dangerous decrease of the radius of the orbit.

References

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